2012

Unsupervised learning of probabilistic grammars

Kewei Tu
Iowa State University

Unsupervised learning of probabilistic grammars

by

Kewei Tu

A dissertation submitted to the graduate faculty

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

Major: Computer Science

Program of Study Committee:

Vasant Honavar, Major Professor

Drena Dobbs

Jack Lutz

Giora Slutzki

Jin Tian

Iowa State University

Ames, Iowa

2012

DEDICATION

I would like to dedicate this thesis to my parents and beloved wife.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ACKNOWLEDGEMENTS

I would like to take this opportunity to express my gratitude to a lot of people without whose support and help I would not have been able to complete this thesis.

First of all, I would like to thank my major professor, Dr. Vasant Honavar, for the advising and support throughout the years of my PhD studies. I want to thank him for giving me the freedom to choose the research topics that I am most interested in while keeping me going in the right direction, and for the guidance on developing my research methodology and career. I also would like to thank my PhD committee members, Drs. Jin Tian, Giora Slutzki, Drena Dobbs and Jack Lutz, for all their help and advice. A special thanks to Dr. Tian for introducing me to probabilistic graphical models and for the seminars and discussions on learning graphical models. I would also like to thank Dr. Alexander Stoytchev for his courses and discussions in my first year of PhD studies, which greatly broadened my views of AI researches and partly motivated my current work.

I would like to thank the former and current students in the AI lab for the helpful discussions and friendship. Thanks to Adrian Silvescu for his thesis work and discussions that partly motivated my current research, to Jie Bao and Feihong Wu for their help during my first few years in the lab, to Oksana Yakhnenko and Cornelia Caragea for the discussions and collaborations in the research of common interest, and to Yasser El-Manzalawy, Rafael Jordan, Fadi Towfic, Neeraj Koul, Li Xue, Jia Tao, Harris Lin, Sangchack Lee, Ngot Bui, Rasna Walia and many others. I also would like to thank fellow graduate students in the department for their friendship and help. A special thanks to Ru He and Yetian Chen for discussions and collaborations in the research of learning graphical models.

I would like to thank my colleagues during my two internships at Microsoft Research Asia, especially Chenxi Lin, Long Zhu and Yuanhao Chen, for their research and discussions with me that partly motivated my work in grammar learning. I would like to thank Prof. Yong Yu,

Dingyi Han, Xixiu Ouyang, Bingkai Lin in the Apex lab of Shanghai Jiaotong University for research collaborations and support.

I also would like to thank all my friends that I made before and during my stay at Ames. I would especially thank Bojian Xu and Song Sun for their tremendous help, support and friendship in my life during the past several years.

Finally, my deepest thanks to my family for the unconditional love and support: thank you for understanding and supporting my choice of pursuing PhD abroad, and for tolerating my staying in school for so long: I am graduating at last :)

During my PhD studies I was supported in part by research assistantships funded by National Science Foundation (IIS 0711356) and the Iowa State University Center for Computational Intelligence, Learning and Discovery, and in part by teaching assistantships in Computer Science Department.

ABSTRACT

Probabilistic grammars define joint probability distributions over sentences and their grammatical structures. They have been used in many areas, such as natural language processing, bioinformatics and pattern recognition, mainly for the purpose of deriving grammatical structures from data (sentences). Unsupervised approaches to learning probabilistic grammars induce a grammar from unannotated sentences, which eliminates the need for manual annotation of grammatical structures that can be laborious and error-prone. In this thesis we study unsupervised learning of probabilistic context-free grammars and probabilistic dependency grammars, both of which are expressive enough for many real-world languages but remain tractable in inference. We investigate three different approaches.

The first approach is a structure search approach for learning probabilistic context-free grammars. It acquires rules of an unknown probabilistic context-free grammar through iterative coherent biclustering of the bigrams in the training corpus. A greedy procedure is used in our approach to add rules from biclusters such that each set of rules being added into the grammar results in the largest increase in the posterior of the grammar given the training corpus. Our experiments on several benchmark datasets show that this approach is competitive with existing methods for unsupervised learning of context-free grammars.

The second approach is a parameter learning approach for learning natural language grammars based on the idea of *unambiguity regularization*. We make the observation that natural language is remarkably unambiguous in the sense that each natural language sentence has a large number of possible parses but only a few of the parses are syntactically valid. We incorporate this prior information into parameter learning by means of posterior regularization. The resulting algorithm family contains classic EM and Viterbi EM, as well as a novel softmax-EM algorithm that can be implemented with a simple and efficient extension to classic EM. Our experiments show that unambiguity regularization improves natural language grammar learning,

and when combined with other techniques our approach achieves the state-of-the-art grammar learning results.

The third approach is grammar learning with a curriculum. A curriculum is a means of presenting training samples in a meaningful order. We introduce the *incremental construction hypothesis* that explains the benefits of a curriculum in learning grammars and offers some useful insights into the design of curricula as well as learning algorithms. We present results of experiments with (a) carefully crafted synthetic data that provide support for our hypothesis and (b) natural language corpus that demonstrate the utility of curricula in unsupervised learning of real-world probabilistic grammars.

CHAPTER 1. Introduction

A grammar is a set of rules that specifies the set of valid sentences of a language as well as the grammatical structures (i.e., parses) of such sentences. The grammar can be used to generate any valid sentence of the language. It can also be used to recognize whether a given sentence is valid, and to derive the grammatical structure of any valid sentence. Aside from their original use in natural language, grammars have been applied in many other areas like programming languages, bioinformatics, and pattern recognition, for the purpose of deriving hidden structures (parses) from data (sentences). For example, in bioinformatics, context-free grammars have been used in predicting RNA secondary structures [Durbin et al. (1998)], where the RNA sequences are the sentences and their secondary structures are the parses. As another example, in pattern recognition, context-sensitive grammars have been applied for object recognition and parsing [Zhu and Mumford (2006)], where the input images are the sentences and the compositional structures of the objects in the images are the parses.

In many real-world application scenarios of grammars, uncertainty is ubiquitous which may arise from the intrinsic ambiguity of the grammars and/or the incompleteness of the observed data. Therefore, probabilistic grammars, which associate probabilities with grammar rules, have been developed to deal with such uncertainty. More formally, a probabilistic grammar defines a joint probability of a sentence and its grammatical structure. By using probabilistic inference, we can recover the grammatical structures from the sentences in a more robust way when uncertainty is present.

Manually constructing a probabilistic grammar for a real-world application usually requires substantial human effort. Machine learning offers a potentially powerful approach to automatically inducing unknown grammars from data (a training corpus). A supervised learning method requires all the sentences in the training corpus to be manually annotated with their grammat-

ical structures. However, such manual annotation process is both laborious and error-prone, and therefore the availability, quality, size and diversity of the annotated training corpora is quite limited. On the other hand, an unsupervised learning method requires only unannotated sentences, making it a more desirable way for learning grammars when annotated corpus is scarce. In this thesis we focus on unsupervised learning of probabilistic grammars.

There are many different types of probabilistic grammars, which can be organized into a hierarchy. At the bottom of the hierarchy, we have Markov models and probabilistic regular grammars (aka. hidden Markov models), which are relatively easy to learn and to do inference with, but have limited expressive power. Higher in the hierarchy, we have probabilistic context-sensitive grammars, which are very expressive and powerful but lead to intractable inference and learning. In this thesis we study two related types of grammars that are in the middle of the hierarchy: probabilistic context-free grammars (PCFG) and probabilistic dependency grammars (DG). They are expressive enough to model complicated languages such as (a significant subset of) natural languages, but remain tractable in inference. There has been a significant amount of work in studying unsupervised learning of these two types of grammars, but there remains much room for improvement.

1.1 Three Types of Approaches to Unsupervised Learning of Probabilistic Grammars

The learning of a probabilistic grammar includes two parts: the learning of the grammar rules (i.e., the structure of the grammar) and the learning of the rule probabilities (i.e., the parameters of the grammar). We can divide existing approaches to unsupervised learning of probabilistic grammars into the following three types.

1. The *structure search* approaches try to find the optimal set of grammar rules. Most of these approaches perform local search with operations on grammar rules, e.g., adding, removing or changing grammar rules. To assign probabilities to the learned grammar rules, some of these approaches make use of a parameter learning approach while others assign the probabilities in a heuristic way.

2. The *parameter learning* approaches assume a fixed set of grammar rules and try to learn their probabilities. Some parameter learning approaches, especially those encouraging parameter sparsity, can also be used to refine the set of grammar rules by removing rules with very small probabilities.

3. The *incremental learning* approaches are meta-algorithms that specify a series of intermediate learning targets which culminate in the actual learning target. These meta-algorithms can utilize either structure search approaches or parameter learning approaches as the subroutine.

The structure search approaches try to solve unsupervised grammar learning as a discrete optimization problem. Because of the difficulty in searching in the super-exponentially large structure space, many structure search approaches rely on heuristics and approximations. In contrast, the parameter learning approaches try to solve unsupervised grammar learning as a continuous optimization problem, which is in general much easier than discrete optimization. Even though a complete search in the parameter space is still infeasible, many well-established approximation techniques can be used to efficiently find a suboptimal solution. As a result, most of the state-of-the-art approaches for unsupervised grammar learning in real-world applications are based on parameter learning. The incremental learning approaches try to set up a series of optimization goals related to the actual optimization goal in order to ease the learning. For some very complicated real-world grammars (e.g., natural language grammars), they may provide a better learning result than the direct application of the structure search or parameter learning approaches.

1.2 Related Work

1.2.1 Unsupervised Grammar Learning

In this section we review previous work related to unsupervised learning of context-free grammars and dependency grammars. There is also a large body of work on learning other types of grammars, e.g., learning regular grammars [de la Higuera (2005); Baum et al. (1970); Teh et al. (2006); Hsu et al. (2009)] and learning more expressive grammars like tree-substitution

grammars [Bod (2006); Johnson et al. (2006); Cohn et al. (2009)], which we will not cover here. We divide our discussion of previous work based on the three types of approaches mentioned earlier.

1.2.1.1 Structure Search Approaches

Some of the previous structure search approaches do not assume the training corpus to be strictly i.i.d. sampled and try to learn a non-probabilistic grammar. EMILE [Adriaans et al. (2000)] constructs from the training corpus a binary table of expressions vs. contexts, and performs biclustering on the table to induce grammar rules that produce strings of terminals; after that, EMILE uses the substitutability heuristic to find high-level grammar rules. ABL [van Zaanen (2000)] employs the substitutability heuristic to group possible constituents to nonterminals, while the approach proposed by Clark (2007) uses the "substitution-graph" heuristic or distributional clustering [Clark (2001)] to induce new nonterminals and grammar rules. Both of these two approaches rely on some heuristic criterion to filter out non-constituents. ADIOS [Solan et al. (2005)] iteratively applies a probabilistic criterion to learn "patterns" (compositions of symbols) and the substitutability heuristic to learn "equivalence classes" (groupings of symbols). GRIDS [Langley and Stromsten (2000)] utilizes two similar operations but relies on beam search to optimize the total description length of the learned grammar and the corpus.

There are also a few structure search approaches that adopt a probabilistic framework. Stolcke and Omohundro (1994) tried to maximize the posterior of the learned grammar by local search with the operations of merging (of existing nonterminals) and chunking (to create new nonterminals from the composition of existing nonterminals). Chen (1995) also tried to find the maximum a posteriori grammar by local search, with two types of operations that both add new rules between existing nonterminals into the grammar. Kurihara and Sato (2006) used the free energy of variational inference as the objective function for local search, with three operations of merging nonterminals, splitting nonterminals and deleting rules.

1.2.1.2 Parameter Learning Approaches

The inside-outside algorithm [Baker (1979); Lari and Young (1990)] is one of the earliest algorithms for learning the parameters of probabilistic context-free grammars. It is a special case of the expectation-maximization (EM) algorithm, which tries to maximize the likelihood of the grammar, making it very likely to overfit the training corpus. Klein and Manning (2004) also used the EM algorithm in learning the dependency grammar, but their approach utilizes a sophisticated initialization grammar which significantly mitigates the local minimum problem of EM. Spitkovsky et al. (2010b) discovered that Viterbi EM, which is a degraded version of EM, can achieve better results in learning natural language grammars. More recent work has adopted the Bayesian framework and introduced various priors into learning. Kurihara and Sato (2004) used a Dirichlet prior over rule probabilities and derived a variational method for learning. Johnson et al. (2007) also used a Dirichlet prior (with less-than-one hyperparameters to encourage parameter sparsity), and they proposed two Markov Chain Monte Carlo methods for learning. Finkel et al. (2007) and Liang et al. (2007) proposed the use of the hierarchical Dirichlet process prior which encourages a smaller grammar size without assuming a fixed number of nonterminals. Cohen et al. (2008) and Cohen and Smith (2009) employed the logistic normal prior to model the correlations between grammar symbols.

Techniques other than probabilistic inference have also been used in parameter learning. Headden et al. (2009) applied linear-interpolation smoothing in learning lexicalized dependency grammars. Gillenwater et al. (2010) incorporated the structural sparsity bias into grammar learning by means of posterior regularization. Daumé (2009) adapted a supervised structured prediction approach for unsupervised use and applied it to unsupervised dependency grammar learning.

1.2.1.3 Incremental Learning Approaches

There exist only a few incremental learning approaches for unsupervised grammar learning. Structural annealing [Smith and Eisner (2006)] gradually decreases the strength of two types of structural bias to guide the iterative learning of dependency grammars. Baby-step [Spitkovsky

et al. (2010a)] starts learning with a training corpus consisting of only length-one sentences, and then adds increasingly longer sentences into the training corpus.

1.2.1.4 Limitations

In spite of the large body of existing work, the performance of unsupervised grammar learning still lags far behind the performance of supervised approaches, especially on real-world data like natural languages, which implies much room for improvement. Moreover, on natural language data, unsupervised learning of context-free grammars (CFG) is much less studied than unsupervised learning of dependency grammars, even though some best-performance natural language parsers are based on CFG (learned in a (semi-)supervised way, e.g., [Petrov et al. (2006)]). This is most likely because a CFG typically contains much more parameters and produces much more possible parses of a sentence than a dependency grammar, which makes CFG much more difficult to learn. A third observation is that almost all the unsupervised learning approaches of dependency grammars only learn unlabeled dependencies, although dependency labels can be very useful in applications like information extraction [Poon and Domingos (2009)]. One possible reason is that adding labels to dependencies dramatically increases the number of parameters as well as the number of possible parses of a sentence, making the learning task much harder.

1.2.2 Supervised Grammar Learning

Supervised grammar learning induces a grammar from a *treebank*, which is a corpus where each sentence is manually parsed by linguists. One can simply count the number of times a production rule is used in the treebank to construct a probabilistic grammar. In natural language parsing, however, a grammar learned in this way (e.g., from the Penn Treebank [Marcus et al. (1993)]) achieves a parsing accuracy well below the current state-of-the-art [Charniak (1996); Petrov et al. (2006)]. The main reason is that while a rule probability is solely conditioned on the left-hand side nonterminal of the rule, the nonterminals used in manual parsing usually do not convey enough information to distinguish different conditions. Therefore many approaches have been developed to augment the treebank grammar, e.g., parent

annotation [Johnson (1998)], nonterminal splitting [Klein and Manning (2003); Petrov et al. (2006); Liang et al. (2007)], lexicalization [Collins (1999); Charniak (1997)].

1.2.3 Theoretical Studies of Grammar Induction

There has been a significant amount of work in the theoretical studies of grammar induction, but the main focus in that field is on regular grammars. For context-free grammars (CFG), it has been shown that CFG is not identifiable in the limit [Gold (1967)]. However, positive results have also been obtained with easier and more realistic settings on some subclasses of CFG (for example, [Clark et al. (2008)]). See [de la Higuera (2005)] for a comprehensive survey.

1.2.4 Other Related Areas

Unsupervised grammar learning is also related to the following research areas.

Structured prediction studies the prediction problem in which the output variables are mutually dependent. Grammar learning can be seen as a special case of structured prediction. Supervised and semi-supervised structured prediction has received substantial attention in recent years and has been applied to grammar learning. On the other hand, unsupervised structured prediction (e.g., [Daumé (2009)]) has received much less attention.

Deep learning tries to construct a deep network consisting of a hierarchy of high level features on top of the inputs. Such deep structures have been found to perform better than traditional shallow learners. Some types of grammars, e.g., probabilistic context-free grammars, can also derive deep structures from their input, in which the variables contain high level information, e.g., syntactic roles of a phrase. Such grammars may be extended for general deep learning (see, for example, [Poon and Domingos (2011)]). Unsupervised learning is essential for deep learning because the deep structure is generally not observable to the learner.

Graphical model structure learning is related to probabilistic grammar learning because probabilistic grammars can be seen as special dynamic graphical models. More specifically, unsupervised learning of a probabilistic grammar can be seen as learning the struc-

ture (and parameters) of a certain type of dynamic graphical models with hidden variables. Indeed, some existing grammar learning algorithms are special cases of graphical model structure learning algorithms.

The cognitive research of first language acquisition studies how human learn their native languages (including the grammars of the languages). Note that human children learn their native language in a largely unsupervised way, in the sense that they learn the language mostly from the speaking of adults, which does not explicitly reveal the underlying grammatical structures.

1.3 Thesis Overview

In this thesis we propose three novel approaches for unsupervised learning of probabilistic grammars.

The first approach is a structure search approach for learning probabilistic context-free grammars [Tu and Honavar (2008)]. It acquires rules of an unknown probabilistic context-free grammar through iterative coherent biclustering of the bigrams in the training corpus. A greedy procedure is used in our approach to add rules from biclusters such that each set of rules being added into the grammar results in the largest increase in the posterior of the grammar given the training corpus. Our experiments on several benchmark datasets show that this approach is competitive with existing methods for unsupervised learning of context-free grammars.

Structure search approaches, however, cannot scale up well to real-world data that is sparse and noisy, e.g., natural language data. In comparison, parameter learning approaches are more scalable, and therefore most of the state-of-the-art algorithms for unsupervised learning of natural language grammars are parameter learning approaches. Our second approach is a parameter learning approach for learning natural language grammars based on the idea of *unambiguity regularization* [Tu and Honavar (2012)]. We make the observation that natural language is remarkably unambiguous in the sense that each natural language sentence has a large number of possible parses but only a few of the parses are syntactically valid. We incorporate this prior information into parameter learning by means of posterior regularization

[Ganchev et al. (2010)]. The resulting algorithm family contains classic EM and Viterbi EM, as well as a novel softmax-EM algorithm that can be implemented with a simple and efficient extension to classic EM. Our experiments show that unambiguity regularization improves natural language grammar learning, and when combined with other techniques our approach achieves the state-of-the-art grammar learning results.

For some very complicated real-world grammars (e.g., natural language grammars), incremental approaches can provide a better learning result than both structure search approaches and parameter learning approaches. So our third approach is an incremental learning approach, namely learning with a curriculum [Tu and Honavar (2011)]. A curriculum is a means of presenting training samples in a meaningful order. We introduce the *incremental construction hypothesis* that explains the benefits of a curriculum in learning grammars and offers some useful insights into the design of curricula as well as learning algorithms. We present results of experiments with (a) carefully crafted synthetic data that provide support for our hypothesis and (b) natural language corpus that demonstrate the utility of curricula in unsupervised learning of real-world probabilistic grammars.

The rest of the thesis is organized as follows.

- In chapter 2, we introduce preliminary concepts and problem definitions.
- In chapter 3, we present our structure search approach for learning probabilistic context-free grammars based on iterative biclustering.
- In chapter 4, we describe our parameter learning approach for learning natural language grammars based on the idea of unambiguity regularization.
- In chapter 5, we study the incremental learning approach that uses curricula.
- In chapter 6, we conclude the thesis with a summary of contributions and directions for future research.

CHAPTER 2. Preliminaries

In this chapter we define the basic concepts and formalize the the problem of unsupervised learning of probabilistic grammars.

2.1 Probabilistic Grammar

A formal grammar is a 4-tuple $\langle \Sigma, N, S, R \rangle$

- Σ is a set of terminal symbols, i.e., the vocabulary of a language
- N is a set of nonterminal symbols (disjoint from Σ)
- $S \in N$ is a start symbol
- R is a set of production rules. Each rule specifies how a string of terminals and/or nonterminals can be rewritten into another string of terminals and/or nonterminals.

A grammar defines valid generative processes of strings in a language, that is, starting from a string containing only the start symbol S and recursively applying the rules in R to rewrite the string until it contains only terminals. This string is called the *sentence*, and the generative process is its *grammatical structure*.

A probabilistic grammar is a grammar with a probability associated to each rule, such that the probabilities of rules with the same left-hand side sum up to 1. In other words, the probability of a grammar rule $\alpha \to \beta$ is the conditional probability of producing the right-hand side β given the left-hand side α. A probabilistic grammar defines a joint probability of a sentence x and its grammatical structure y:

$$P(x, y|G) = \prod_{r \in R} {\theta_r}^{f_r(x,y)}$$

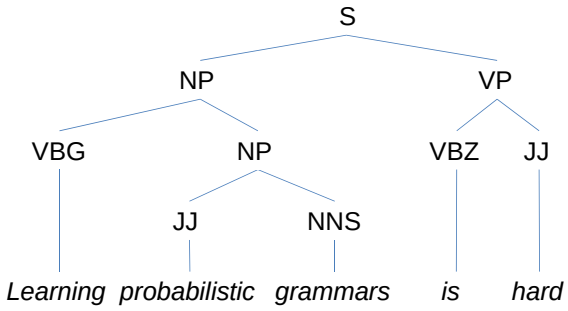


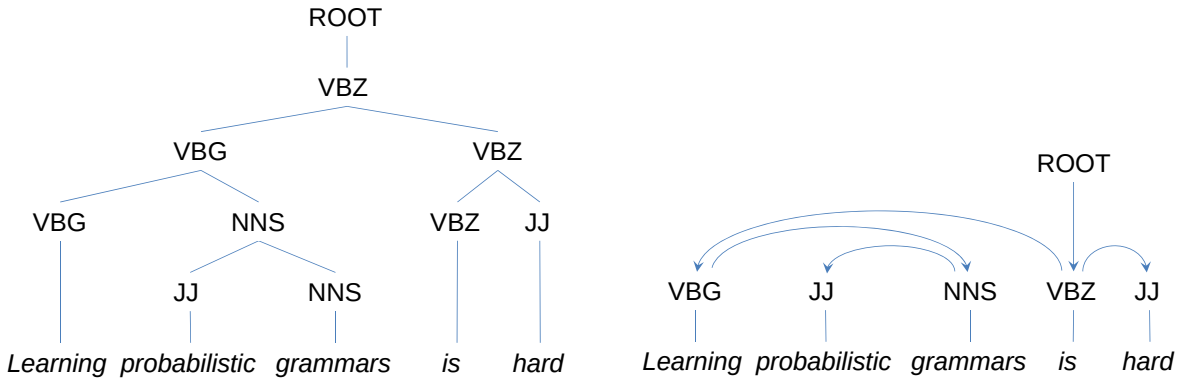
Figure 2.1 A natural language sentence and its grammatical structure generated by a PCFG. The whole tree structure is the grammatical structure, and the leaf nodes constitute the sentence.

where G is the probabilistic grammar, θ_r is the probability of rule r in G, and $f_r(x, y)$ is the number of times rule r is used in the generative process of x as specified by y.

In this thesis we will focus on two types of probabilistic grammars: probabilistic context-free grammars and probabilistic dependency grammars. Both of these two types of probabilistic grammars are expressive enough to represent many real world structures such as natural languages, while still remain computationally tractable in inference and learning.

A probabilistic context-free grammar (PCFG) is a probabilistic grammar such that for any of its grammar rules, the left-hand side of the rule is a single nonterminal. In other words, every rule in a PCFG must take the form of $A \to \gamma$, where A is a nonterminal and γ is a string of terminals and/or nonterminals. It is easy to see that the grammatical structure generated by a PCFG is a tree. Figure 2.1 shows an example English sentence and its grammatical structure specified by a PCFG.

A (probabilistic) dependency grammar is a (probabilistic) grammar that requires its grammar rules to take the form of $ROOT \to A$, $A \to AB$, $A \to BA$ or $A \to a$, where $ROOT$ is the start symbol, A and B are nonterminals, and a is a terminal. So dependency grammars are a subclass of context-free grammars. Figure 2.2(a) shows the grammatical structure of the example sentence specified by a dependency grammar. Equivalently, we can represent the grammatical structure specified by a dependency grammar using a set of *dependencies*, as shown in Figure 2.2(b). Each dependency is a directed arc between two nonterminals.



(a) The parse tree representation (b) The dependency representation

Figure 2.2 The grammatical structure generated by a dependency grammar.

There are several variants of dependency grammars. In this thesis we use a variant named dependency model with valence (DMV) [Klein and Manning (2004)]. DMV extends dependency grammars by introducing an additional set of rules that determine whether a new dependency should be generated from a nonterminal, conditioned on the nonterminal symbol, the direction of the dependency (i.e., left vs. right) and the adjacency (whether a dependency has already been generated from the nonterminal in that direction). So take the sentence in Figure 2.2 for example, with a probabilistic DMV the probability of VBZ generating VBG and JJ in the dependency structure is

$$P(\neg\text{stop}|\text{VBZ}, direction = \text{left}, adjacency = \text{false})\ P(\text{VBG}|\text{VBZ}, direction = \text{left})$$
$$\times P(\text{stop}|\text{VBZ}, direction = \text{left}, adjacency = \text{true})$$
$$\times P(\neg\text{stop}|\text{VBZ}, direction = \text{right}, adjacency = \text{false})\ P(\text{JJ}|\text{VBZ}, direction = \text{right})$$
$$\times P(\text{stop}|\text{VBZ}, direction = \text{right}, adjacency = \text{true})$$

Klein and Manning (2004) have shown that adding this set of grammar rules leads to better models of natural language and therefore better learning results in natural language grammar learning. It is easy to see that DMV is still a subclass of context-free grammars.

Most of the state-of-the-art approaches in unsupervised natural language grammar learning try to learn a DMV or extensions of DMV. This is probably because compared with PCFG, DMV contains less nonterminals and much less valid grammar rules, which makes it much easier

to learn. On the other hand, as can be seen in Figure 2.1 and 2.2, the grammatical structure generated by a PCFG does provide more information than the grammatical structure generated by a dependency grammar, i.e., a PCFG uses a different set of nonterminals for non-leaf nodes in the parse tree which can be used to convey additional linguistic information (e.g., the type of phrase). In addition, since dependency grammars are a subclass of context-free grammars, there may exist linguistic phenomena that can be modeled by a PCFG but not by a dependency grammar. Therefore, some state-of-the-art natural language parsers are based on PCFG (e.g., the Berkeley parser [Petrov et al. (2006)], which is learned in a supervised way).

2.2 Unsupervised Learning of Probabilistic Grammars

Unsupervised grammar learning tries to learn a grammar from a set of unannotated sentences (i.e., with no information of the grammatical structures). These sentences are usually assumed to be generated independently from the same probabilistic grammar (the i.i.d. assumption). Formally, given a set of sentences $D = \{x_i\}$, we want to find

$$G^* = \arg\max_G P(G|D)$$

Unsupervised grammar learning saves the substantial cost incurred by manually annotating the grammatical structures of the training sentences. It also avoids potential limitations of the annotations, e.g., the size and coverage of the annotated corpus, and the errors and biases of the annotations. On the other hand, with the grammatical structures of the training sentences hidden from the learner, it becomes very difficult to learn an accurate grammar.

Note that this type of learning is called "unsupervised" in the sense that we want to use the grammar to derive the grammatical structures of sentences while the structure information is not available in the training data. If, on the other hand, the goal is to distinguish grammatical sentences from ungrammatical ones, then this learning scenario can be seen as a one-class classification problem [Tax (2001)], because only grammatical sentences are presented in the training set.

As introduced in Chapter 1, the approaches to unsupervised learning of probabilistic grammars can be divided into three types: structure search, parameter learning and incremental

learning. Here we give a more formal definition of these three types of approaches.

Note that a probabilistic grammar G can be represented by two variables: the set of grammar rules R (i.e., the grammar structure) and the rule probabilities Θ (i.e., the grammar parameters). Structure search approaches learn the grammar structure along with the grammar parameters.

$$G^* = \arg\max_{(R,\Theta)} P(R, \Theta | D)$$

Parameter learning approaches assume a fixed grammar structure R_0 (which is usually assumed to include all possible grammar rules), and try to learn the grammar parameters.

$$G^* = \left(R_0, \arg\max_{\Theta} P(\Theta | D, R_0) \right)$$

Incremental approaches specify a series of intermediate learning targets $\langle G_1, G_2, \ldots, G_n \rangle$ which culminate in the actual learning target G^*. The intermediate learning targets are usually specified implicitly by modifying the objective function, e.g., imposing additional constraints, changing the hyperparameters of the priors, or weighting the training sentences. Typically we require that each intermediate target is closer to the final target G^* than any previous intermediate target:

$$\forall i < j, \quad d(G_i, G^*) \geq d(G_j, G^*)$$

where d is some kind of distance measure. The learner iteratively pursues each intermediate target based on the result of pursuing the previous intermediate target, until the final target G^* is presented to the learner.

CHAPTER 3. A Structure Search Approach Based on Iterative Biclustering

The structure search approaches for learning probabilistic grammars try to find the optimal set of grammar rules. Most of these approaches use local search with operations on grammar rules, e.g., adding, removing or changing grammar rules. To assign the grammar rule probabilities, some of these approaches make use of a parameter learning approach while others assign the probabilities in a heuristic way.

This chapter presents a structure search approach named PCFG-BCL for unsupervised learning of probabilistic context-free grammars (PCFG). The algorithm acquires rules of an unknown PCFG through iterative biclustering of bigrams in the training corpus. Our analysis shows that this procedure uses a greedy approach to adding rules such that each set of rules that is added to the grammar results in the largest increase in the posterior of the grammar given the training corpus. Results of our experiments on several benchmark datasets show that PCFG-BCL is competitive with existing methods for unsupervised CFG learning.

3.1 Introduction

In this chapter we propose PCFG-BCL, a new structure search algorithm for unsupervised learning of probabilistic context-free grammars (PCFG). The proposed algorithm uses (distributional) biclustering to group symbols into non-terminals. This is a more natural and robust alternative to the more widely used substitutability heuristic or distributional clustering, especially in the presence of ambiguity, e.g., when a symbol can be reduced to different nonterminals in different contexts, or when a context can contain symbols of different nonterminals, as illustrated in [Adriaans et al. (2000)]. PCFG-BCL can be understood within a Bayesian structure

search framework. Specifically, it uses a greedy approach to adding rules to a partially constructed grammar, choosing at each step a set of rules that yields the largest possible increase in the posterior of the grammar given the training corpus. The Bayesian framework also supports an ensemble approach to PCFG learning by effectively *combining* multiple candidate grammars. Results of our experiments on several benchmark datasets show that the proposed algorithm is competitive with other methods for learning CFG from positive samples.

The rest of the chapter is organized as follows. Section 3.2 introduces the representation of PCFG used in PCFG-BCL. Section 3.3 describes the key ideas behind PCFG-BCL. Section 3.4 presents the complete algorithm and some implementation details. Section 3.5 presents the results of experiments. Section 3.6 concludes with a summary and a brief discussion of related work.

3.2 Grammar Representation

It is well-known that any CFG can be transformed into the Chomsky normal form (CNF), which only has two types of rules: $A \to BC$ or $A \to a$. Because a PCFG is simply a CFG with a probability associated with each rule, it is easy to transform a PCFG into a probabilistic version of CNF.

To simplify the explanation of our algorithm, we make use of the fact that a CNF grammar can be represented in an AND-OR form containing three types of symbols, i.e., AND, OR, and terminals. An AND symbol appears on the left-hand side of exactly one grammar rule, and on the right-hand side of that rule there are exactly two OR symbols. An OR symbol appears on the left-hand side of one or more rules, each of which has only one symbol on the right-hand side, either an AND symbol or a terminal. A multinomial distribution can be assigned to the set of rules of an OR symbol, defining the probability of each rule being chosen. An example is shown below (with rules probabilities in the parentheses).

CNF	**The AND-OR Form**
$S \to a$ (0.4) $\mid AB$ (0.6)	$\text{OR}_S \to a$ (0.4) $\mid \text{AND}_{AB}$ (0.6)
$A \to a$ (1.0)	$\text{AND}_{AB} \to \text{OR}_A\text{OR}_B$
$B \to b_1$ (0.2) $\mid b_2$ (0.5) $\mid b_3$ (0.3)	$\text{OR}_A \to a$ (1.0)
	$\text{OR}_B \to b_1$ (0.2) $\mid b_2$ (0.5) $\mid b_3$ (0.3)

It is easy to show that a CNF grammar in the AND-OR form can be divided into a set of *AND-OR groups* plus the start rules (rules with the start symbol on the left-hand side). Each AND-OR group contains an AND symbol N, two OR symbols A and B such that $N \to AB$, and all the grammar rules that have one of these three symbols on the left-hand side. In the above example, there is one such AND-OR group, i.e., AND_{AB}, OR_A, OR_B and the corresponding rules (the last three lines). Note that there is a bijection between the AND symbols and the groups; but an OR symbol may appear in multiple groups. We may simply make identical copies of such OR symbols to eliminate overlap between groups.

3.3 Main Ideas

PCFG-BCL is designed to learn a PCFG using its CNF representation in the AND-OR form. Sentences in the training corpus are assumed to be sampled from an unknown PCFG under the i.i.d. (independent and identically distributed) assumption.

Starting from only terminals, PCFG-BCL iteratively adds new symbols and rules to the grammar. At each iteration, it first learns a new AND-OR group by biclustering, as explained in Section 3.3.1. Once a group is learned, it tries to find rules that attach the newly learned AND symbol to existing OR symbols, as discussed in Section 3.3.2. This second step is needed because the first step alone is not sufficient for learning such rules. In both steps, once a new set of rules are learned, the corpus is *reduced* using the new rules, so that subsequent learning can be carried out on top of the existing learning result. These two steps are repeated until no further rule can be learned. Then start rules are added to the learned grammar in a postprocessing step (Section 3.3.3). Since any CNF grammar can be represented in the form of a set of AND-OR groups and a set of start rules, these three steps are capable, in principle, of constructing any CNF grammar.

We will show later that the first two steps of PCFG-BCL outlined above attempt to find rules that yield the greatest increase in the posterior probability of the grammar given the training corpus. Thus, PCFG-BCL performs a local search over the space of grammars using the posterior as the objective function.

3.3.1 Learning a New AND-OR Group by Biclustering

3.3.1.1 Intuition.

In order to show what it means to learn a new AND-OR group, it is helpful to construct a table T, where each row or column represents a symbol appearing in the corpus, and the cell at row x and column y records the number of times the pair xy appears in the corpus. Because the corpus might have been partially reduced in previous iterations, a row or column in T may represent either a terminal or a nonterminal.

Since we assume the corpus is generated by a CNF grammar, there must be some symbol pairs in the corpus that are generated from AND symbols of the target grammar. Let N be such an AND symbol, and let A, B be the two OR symbols such that $N \to AB$. The set $\{x|A \to x\}$ corresponds to a set of rows in the table T, and the set $\{y|B \to y\}$ corresponds to a set of columns in T. Therefore, the AND-OR group that contains N, A and B is represented by a *bicluster* [Madeira and Oliveira (2004)] (i.e., a submatrix) in T, and each pair xy in this bicluster can be reduced to N. See Fig.3.1 (a), (b) for an example, where the AND-OR group shown in Fig.3.1(a) corresponds to the bicluster shown in Fig.3.1(b).

Further, since we assume the target grammar is a PCFG, we have two multinomial distributions defined on A and B respectively that independently determine the symbols generated from A and B. Because the corpus is assumed to be generated by this PCFG, it is easy to prove that the resulting bicluster must be *multiplicatively coherent* [Madeira and Oliveira (2004)], i.e., it satisfies the following condition:

$$\frac{a_{ik}}{a_{jk}} = \frac{a_{il}}{a_{jl}} \quad \text{for any two rows } i, j \text{ and two columns } k, l \tag{3.1}$$

where a_{xy} is the cell value at row x ($x = i, j$) and column y ($y = k, l$).

$\text{AND}_{NP} \rightarrow \text{OR}_{Det}\text{OR}_{N}$
$\text{OR}_{Det} \rightarrow$ the(0.67) | a(0.33)
$\text{OR}_{N} \rightarrow$ circle(0.2)
| triangle(0.3) | square(0.5)

(a) An AND-OR group (with rule probabilities in the parentheses)

	is	**circle**	**triangle**	**square**	the	⋯
below					8	
above					10	
the		24	36	60		
a		12	18	30		
circle	4					
triangle	6					
⋮						

(b) A part of the table T and the bicluster that represents the AND-OR group. Zero cells are left blank.

	⋯ covers (.)	⋯ touches (.)	⋯ is above (.)	⋯ is below (.)	(.) rolls.	(.) bounces.	⋯
(a, circle)	1	2	1	1	0	0	
(a, triangle)	1	2	1	3	2	1	
(a, square)	3	4	2	4	4	1	⋯
(the, circle)	2	3	1	3	2	1	
(the, triangle)	3	5	2	5	4	2	
(the, square)	5	8	4	8	7	3	

(c) A part of the expression-context matrix of the bicluster

Figure 3.1 Example: a bicluster and its expression-context matrix

Given a bicluster in T, we can construct an *expression-context matrix*, in which the rows represent the set of symbol pairs (expressions) in the bicluster, the columns represent all the contexts in which these symbol pairs appear, and the value in each cell denotes the number of times the corresponding expression-context combination appears in the corpus (see Fig.3.1(c) for an example). Because the target grammar is context-free, if a bicluster represents an AND-OR group of the target grammar, then the choice of the symbol pair is independent of its context and thus the resulting expression-context matrix should also be multiplicatively coherent, i.e., it must satisfy Eq.3.1.

The preceding discussion suggests an intuitive approach to learning a new AND-OR group: first find a bicluster of T that is multiplicatively coherent and has a multiplicatively coherent expression-context matrix, and then construct an AND-OR group from it. The probabilities associated with the grammar rules can be estimated from the statistics of the bicluster. For example, if we find that the bicluster shown in Fig.3.1(b) and its expression-context matrix shown in Fig.3.1(c) are both multiplicatively coherent, we can learn an AND-OR group as shown in Fig.3.1(a).

3.3.1.2 Probabilistic Analysis.

We now present an analysis of the intuitive idea outlined above within a probabilistic framework. Consider a trivial initial grammar where the start symbol directly generates each sentence of the corpus with equal probability. We can calculate how the likelihood of the corpus given the grammar is changed by extracting a bicluster and learning a new AND-OR group as described above.

Suppose we extract a bicluster BC and add to the grammar an AND-OR group with an AND symbol N and two OR symbols A and B. Suppose there is a sentence d containing a symbol pair xy that is in BC. First, since xy is reduced to N after this learning process, the likelihood of d is reduced by a factor of $P(N \to xy|N) = P(A \to x|A) \times P(B \to y|B)$. Second, the reduction may make some other sentences in the corpus become identical to d, resulting in a corresponding increase in the likelihood. Suppose the sentence d is represented by row p and column q in the expression-context matrix of BC, then this second factor is exactly the ratio of the sum of column q to the value of cell pq, because before the reduction only those sentences represented by cell pq are equivalent to d, and after the reduction the sentences in the entire column become equivalent (the same context plus the same expression N).

Let $LG(BC)$ be the likelihood gain resulting from extraction of BC; let G_k and G_{k+1} be the grammars before and after extraction of BC, D be the training corpus; in the bicluster BC, let A denote the set of rows, B the set of columns, r_x the sum of entries in row x, c_y the sum of entries in column y, s the sum over all the entries in BC, and a_{xy} the value of cell xy; in the expression-context matrix of BC, let EC-row denote the set of rows, EC-col the set of columns, r'_p the sum of entries in row p, c'_q the sum of entries in column q, s' the sum of all the entries in the matrix, and $EC(p,q)$ or a'_{pq} the value of cell pq. With a little abuse of notation we denote the context of a symbol pair xy in a sentence d by $d-$"xy". We can now calculate the likelihood gain as follows:

$$\begin{aligned} LG(BC) &= \frac{P(D|G_{k+1})}{P(D|G_k)} = \prod_{d \in D} \frac{P(d|G_{k+1})}{P(d|G_k)} \\ &= \prod_{x \in A,\ y \in B,\ xy \text{ appears in } d \in D} P(x|A)P(y|B)\frac{\sum_{p \in \text{EC-row}} EC(p, d - \text{“xy”})}{EC(\text{“xy”}, d - \text{“xy”})} \end{aligned}$$

$$= \prod_{x \in A} P(x|A)^{r_x} \prod_{y \in B} P(y|B)^{c_y} \frac{\prod_{q \in \text{EC-col}} {c'_q}^{c'_q}}{\prod_{\substack{p \in \text{EC-row} \\ q \in \text{EC-col}}} {a'_{pq}}^{a'_{pq}}}$$

It can be shown that, the likelihood gain is maximized by setting:

$$P(x|A) = \frac{r_x}{s} \qquad\qquad P(y|B) = \frac{c_y}{s}$$

Substituting this into the likelihood gain formula, we get

$$\begin{aligned} \max_{P_r} LG(BC) &= \prod_{x \in A} \left(\frac{r_x}{s}\right)^{r_x} \prod_{y \in B} \left(\frac{c_y}{s}\right)^{c_y} \frac{\prod_{q \in \text{EC-col}} {c'_q}^{c'_q}}{\prod_{\substack{p \in \text{EC-row} \\ q \in \text{EC-col}}} {a'_{pq}}^{a'_{pq}}} \\ &= \frac{\prod_{x \in A} {r_x}^{r_x} \prod_{y \in B} {c_y}^{c_y}}{s^{2s}} \times \frac{\prod_{q \in \text{EC-col}} {c'_q}^{c'_q}}{\prod_{\substack{p \in \text{EC-row} \\ q \in \text{EC-col}}} {a'_{pq}}^{a'_{pq}}} \end{aligned}$$

where P_r represents the set of grammar rule probabilities. Notice that $s = s'$ and $a_{xy} = r'_p$ (where row p of the expression-context matrix represents the symbol pair xy). Thus we have

$$\max_{P_r} LG(BC) = \frac{\prod_{x \in A} {r_x}^{r_x} \prod_{y \in B} {c_y}^{c_y}}{s^s \prod_{\substack{x \in A \\ y \in B}} {a_{xy}}^{a_{xy}}} \times \frac{\prod_{p \in \text{EC-row}} {r'_p}^{r'_p} \prod_{q \in \text{EC-col}} {c'_q}^{c'_q}}{{s'}^{s'} \prod_{\substack{p \in \text{EC-row} \\ q \in \text{EC-col}}} {a'_{pq}}^{a'_{pq}}}$$

The two factors in the righthand side are of the same form, one for the bicluster and one for the expression-context matrix. This form of formula actually measures the multiplicative coherence of the underlying matrix (in a slightly different way from Eq.18 of [Madeira and Oliveira (2004)]), which is maximized when the matrix is perfectly coherent. Therefore, we see that when extracting a bicluster (with the new grammar rule probabilities set to the optimal values), the likelihood gain is the product of the multiplicative coherence of the bicluster and its expression-context matrix, and that the maximal gain in likelihood is obtained when both the bicluster and its expression-context matrix are perfectly multiplicatively coherent. This validates the intuitive approach in the previous subsection. More derivation details can be found in Appendix A.

It must be noted however, in learning from data, simply maximizing the likelihood can result in a learned model that overfits the training data and hence generalizes poorly on data unseen during training. In our setting, maximizing the likelihood is equivalent to finding the most coherent biclusters. This can result in a proliferation of small biclusters and hence grammar rules that encode highly specific patterns appearing in the training corpus. Hence learning

algorithms typically have to trade off the complexity of the model against the quality of fit on the training data. We achieve this by choosing the prior $P(G) = 2^{-DL(G)}$ over the set of candidate grammars, where $DL(G)$ is the description length of the grammar G. This prior penalizes more complex grammars, as complex grammars are more likely to overfit the training corpus.

Formally, the logarithm of the gain in posterior as a result of extracting an AND-OR group from a bicluster and updating the grammar from G_k to G_{k+1} (assuming the probabilities associated with the grammar rules are set to their optimal values) is given by:

$$\begin{aligned}
\max_{P_r} LPG(BC) &= \max_{P_r} \log \frac{P(G_{k+1}|D)}{P(G_k|D)} \\
&= \left(\sum_{x \in A} r_x \log r_x + \sum_{y \in B} c_y \log c_y - s \log s - \sum_{x \in A, y \in B} a_{xy} \log a_{xy} \right) \\
&\quad + \left(\sum_{p \in \text{EC-row}} r'_p \log r'_p + \sum_{q \in \text{EC-col}} c'_q \log c'_q - s' \log s' - \sum_{\substack{p \in \text{EC-row} \\ q \in \text{EC-col}}} a'_{pq} \log a'_{pq} \right) \\
&\quad + \alpha \left(4 \sum_{x \in A, y \in B} a_{xy} - 2|A| - 2|B| - 8 \right)
\end{aligned} \tag{3.2}$$

where $LPG(BC)$ denotes the logarithmic posterior gain resulting from extraction of the bicluster BC; α is a parameter in the prior that specifies how much the prior favors compact grammars, and hence it controls the tradeoff between the complexity of the learned grammar and the quality of fit on the training corpus. Note that the first two terms in this formula correspond to the gain in log likelihood (as shown earlier). The third term is the logarithmic prior gain, biasing the algorithm to favor large biclusters and hence compact grammars (see Appendix A for details).

3.3.2 Attaching a New AND Symbol under Existing OR Symbols

3.3.2.1 Intuition.

For a new AND symbol N learned in the first step, there may exist one or more OR symbols in the current partially learned grammar, such that for each of them (denoted by O), there is a rule $O \rightarrow N$ in the target grammar. Such rules cannot be acquired by extracting biclusters as

described above: When O is introduced into the grammar, N simply does not exist in the table T, and when N is introduced, it only appears in a rule of the form $N \to AB$. Hence, we need a strategy for discovering such OR symbols and adding the corresponding rules to the grammar. Note that, if there are recursive rules in the grammar, they are learned in this step. This is because the first step establishes a partial order among the symbols, and only by this step can we connect nonterminals to form cycles and thus introduce recursions into the grammar.

Consider an OR symbol O that was introduced into the grammar as part of an AND-OR group obtained by extracting a bicluster BC. Let M be the AND symbol and P the other OR symbol in the group, such that $M \to OP$. So O corresponds to the set of rows and P corresponds to the set of columns of BC.

If $O \to N$, and if we add to BC a new row for N, where each cell records the number of appearances of Nx (for all x s.t. $P \to x$) in the corpus, then the expanded bicluster should be multiplicatively coherent, for the same reason that BC was multiplicatively coherent. The new row N in BC results in a set of new rows in the expression-context matrix. This expanded expression-context matrix should be multiplicatively coherent for the same reason that the expression-context matrix of BC was multiplicatively coherent. The situation is similar when we have $M \to PO$ instead of $M \to OP$ (thus a new *column* is added to BC when adding the rule $O \to N$). An example is shown in Fig.3.2.

Thus, if we can find an OR symbol O such that the expanded bicluster and the corresponding expanded expression-context matrix are both multiplicatively coherent, we should add the rule $O \to N$ to the grammar.

3.3.2.2 Probabilistic Analysis.

The effect of attaching a new AND symbol under existing OR symbols can be understood within a probabilistic framework. Let $\widetilde{BC}$ be a *derived* bicluster, which has the same rows and columns as BC, but the values in its cells correspond to the expected numbers of appearances of the symbol pairs when applying the current grammar to expand the current partially reduced corpus. $\widetilde{BC}$ can be constructed by traversing all the AND symbols that M can be directly or indirectly reduced to in the current grammar. $\widetilde{BC}$ is close to BC if for all the AND symbols

AND → $OR_1 OR_2$
OR_1 → big (0.6) | old (0.4)
OR_2 → dog (0.6) | cat (0.4)
New rule: OR_2 → AND

(a) An existing AND-OR group and a proposed new rule

	dog	cat	AND
big	27	18	15
old	18	12	10

(b) The bicluster and its expansion (a new column)

	the (.) slept.	the big (.) slept.	the old (.) slept.	the old big (.) slept.	··· heard the (.)	··· heard the old (.)	···
(old, dog)	6	1	1	0	3	1	···
(big, dog)	9	2	1	1	4	1	
(old, cat)	4	1	0	0	2	1	
(big, cat)	6	1	1	0	4	1	
(old, AND)	3	1	0	0	2	1	···
(big, AND)	5	1	1	0	2	1	

(c) The expression-context matrix and its expansion

Figure 3.2 An example of adding a new rule that attaches a new AND under an existing OR. Here the new AND is attached under one of its own OR symbols, forming a self-recursion.

involved in the construction, their corresponding biclusters and expression-context matrices are approximately multiplicatively coherent, a condition that is ensured in our algorithm. Let $\widetilde{BC}'$ be the expanded derived bicluster that contains both $\widetilde{BC}$ and the new row or column for N. It can be shown that the likelihood gain of adding $O \rightarrow N$ is approximately the likelihood gain of extracting $\widetilde{BC}'$, which, as shown in Section 3.3.1, is equal to the product of the multiplicative coherence of $\widetilde{BC}'$ and its expression-context matrix (when the optimal new rule probabilities are assigned that maximize the likelihood gain). Thus it validates the intuitive approach in the previous subsection. See Appendix A for details.

As before, we need to incorporate the effect of the prior into the above analysis. So we search for existing OR symbols that result in maximal posterior gains exceeding a user-specified threshold. The maximal posterior gain is approximated by the following formula.

$$\max_{P_r} \log \frac{P(G_{k+1}|D)}{P(G_k|D)} \approx \max_{P_r} LPG(\widetilde{BC}') - \max_{P_r} LPG(\widetilde{BC}) \tag{3.3}$$

where P_r is the set of new grammar rule probabilities, G_k and G_{k+1} is the grammar before and after adding the new rule, D is the training corpus, $LPG()$ is defined in Eq.3.2. Please see Appendix A for the details.

3.3.3 Postprocessing

The two steps described above are repeated until no further rule can be learned. Since we reduce the corpus after each step, in an ideal scenario, upon termination of this process the corpus is fully reduced, i.e., each sentence is represented by a single symbol, either an AND symbol or a terminal. However, in practice there may still exist sentences in the corpus containing more than one symbol, either because we have applied the wrong grammar rules to reduce them, or because we have failed to learn the correct rules that are needed to reduce them.

At this stage, the learned grammar is almost complete, and we only need to add the start symbol S (which is an OR symbol) and start rules. We traverse the whole corpus: In the case of a fully reduced sentence that is reduced to a symbol x, we add $S \to x$ to the grammar if such a rule is not already in the grammar (the probability associated with the rule can be estimated by the fraction of sentences in the corpus that are reduced to x). In the case of a sentence that is not fully reduced, we can re-parse it using the learned grammar and attempt to fully reduce it, or we can simply discard it as if it was the result of noise in the training corpus.

3.4 Algorithm and Implementation

The complete algorithm is presented in Algorithm 1, and the three steps are shown in Algorithm 2 to 4 respectively. Algorithm 2 describes the "learning by biclustering" step (Section 3.3.1). Algorithm 3 describes the "attaching" step (Section 3.3.2), where we use a greedy solution, i.e., whenever we find a good enough OR symbol, we learn the corresponding new rule. In both Algorithm 2 and 3, a *valid* bicluster refers to a bicluster where the multiplicative coherence of the bicluster and that of its expression-context matrix both exceed a threshold δ. This corresponds to the heuristic discussed in the "intuition" subsections in Section 3.3, and it is used here as an additional constraint in the posterior-guided search. Algorithm 4 describes the postprocessing step (Section 3.3.3), wherein to keep things simple, sentences not fully reduced are discarded.

Algorithm 1 PCFG-BCL: PCFG Learning by Iterative Biclustering

Input: a corpus C
Output: a CNF grammar in the AND-OR form

create an empty grammar G
create a table T of the number of appearances of each symbol pair in C
repeat
    G, C, T, $N \Leftarrow$ LearningByBiclustering(G, C, T)
    G, C, $T \Leftarrow$ Attaching(N, G, C, T)
until no further rule can be learned
$G \Leftarrow$ Postprocessing(G, C)
return G

Algorithm 2 LearningByBiclustering(G, C, T)

Input: the grammar G, the corpus C, the table T
Output: the updated G, C, T; the new AND symbol N

find the valid bicluster Bc in T that leads to the maximal posterior gain (Eq.3.2)
create an AND symbol N and two OR symbols A, B
for all row x of Bc **do**
    add $A \to x$ to G, with the row sum as the rule weight
for all column y of Bc **do**
    add $B \to y$ to G, with the column sum as the rule weight
add $N \to AB$ to G
in C, reduce all the appearances of all the symbol pairs in Bc to N
update T according to the reduction
return G, C, T, N

Algorithm 3 Attaching(N, G, C, T)

Input: an AND symbol N, the grammar G, the corpus C, the table T
Output: the updated G, C, T

for each OR symbol O in G **do**
    if O leads to a valid expanded bicluster as well as a posterior gain (Eq.3.3) larger than a threshold **then**
        add $O \to N$ to G
        maximally reduce all the related sentences in C
        update T according to the reduction
return G, C, T

Algorithm 4 Postprocessing(G, C)

Input: the grammar G, the corpus C
Output: the updated G
1: create an OR symbol S
2: **for** each sentence s in C **do**
3: **if** s is fully reduced to a single symbol x **then**
4: add $S \rightarrow x$ to G, or if the rule already exists, increase its weight by 1
5: **return** G

3.4.1 Implementation Issues

In the "learning by biclustering" step we need to find the bicluster in T that leads to the maximal posterior gain. However, finding the optimal bicluster is computationally intractable [Madeira and Oliveira (2004)]. In our current implementation, we use stochastic hill-climbing to find only a fixed number of biclusters, from which the one with the highest posterior gain is chosen. This method is not guaranteed to find the optimal bicluster when there are more biclusters in the table than the fixed number of biclusters considered. In practice, however, we find that if there are many biclusters, often it is the case that several of them are more or less equally optimal and our implementation is very likely to find one of them.

Constructing the expression-context matrix becomes time-consuming when the average context length is long. Moreover, when the training corpus is not large enough, long contexts often result in rather sparse expression-context matrices. Hence, in our implementation we only check context of a fixed size (by default, only the immediate left and immediate right neighbors). It can be shown that this choice leads to a matrix whose coherence is no lower than that of the true expression-context matrix, and hence may overestimate the posterior gain.

3.4.2 Grammar Selection and Averaging

Because we use stochastic hill-climbing with random start points to do biclustering, our current implementation can produce different grammars in different runs. Since we calculate the posterior gain in each step of the algorithm, for each learned grammar an overall posterior gain can be obtained, which is proportional to the actual posterior. We can use the posterior gain to evaluate different grammars and perform model selection or model averaging, which

Grammar Name	Size (in CNF)	Recursion	Source
Baseline	12 Terminals, 9 Nonterminals, 17 Rules	No	Boogie [Stolcke (1993)]
Num-agr	19 Terminals, 15 Nonterminals, 30 Rules	No	Boogie [Stolcke (1993)]
Langley1	9 Terminals, 9 Nonterminals, 18 Rules	Yes	Boogie [Stolcke (1993)]
Langley2	8 Terminals, 9 Nonterminals, 14 Rules	Yes	Boogie [Stolcke (1993)]
Emile2k	29 Terminals, 15 Nonterminals, 42 Rules	Yes	EMILE [Adriaans et al. (2000)]
TA1	47 Terminals, 66 Nonterminals, 113 Rules	Yes	ADIOS [Solan et al. (2005)]

Table 3.1 The CFGs used in the evaluation.

usually leads to better performance than using a single grammar.

To perform model selection, we run the algorithm multiple times and return the grammar that has the largest posterior gain. To perform model averaging, we run the algorithm multiple times and obtain a set of learned grammars. Given a sentence to be parsed, in the spirit of Bayesian model averaging, we parse the sentence using each of the grammars and use a weighted vote to accept or reject it, where the weight of each grammar is its posterior gain. To generate a new sentence, we select a grammar in the set with the probability proportional to its weight, and generate a sentence using that grammar; then we parse the sentence as described above, and output it if it's accepted, or start over if it is rejected.

3.5 Experiments

A set of PCFGs obtained from available synthetic, English-like CFGs were used in our evaluation, as listed in Table 3.1. The CFGs were converted into CNF with uniform probabilities assigned to the grammar rules. Training corpora were then generated from the resulting grammars. We compared PCFG-BCL with EMILE [Adriaans et al. (2000)] and ADIOS [Solan et al. (2005)]. Both EMILE and ADIOS produce a CFG from a training corpus, so we again assigned uniform distributions to the rules of the learned CFG in order to evaluate them.

We evaluated our algorithm by comparing the learned grammar with the target grammar on the basis of *weak generative capacity*. That is, we compare the language of the learned grammar with that of the target grammar in terms of *precision* (the percentage of sentences generated by the learned grammar that are accepted by the target grammar), *recall* (the percentage of sentences generated by the target grammar that are accepted by the learned grammar), and *F-score* (the harmonic mean of precision and recall). To estimate precision and recall, 200

Grammar Name	PCFG-BCL			EMILE			ADIOS		
	P	R	F	P	R	F	P	R	F
Baseline (100)	100 (0)	100 (0)	100 (0)	100 (0)	100 (0)	100 (0)	100 (0)	99 (2)	99 (1)
Num-agr (100)	100 (0)	100 (0)	100 (0)	50 (4)	100 (0)	67 (3)	100 (0)	92 (6)	96 (3)
Langley1 (100)	100 (0)	100 (0)	100 (0)	100 (0)	99 (1)	99 (1)	99 (3)	94 (4)	96 (2)
Langley2 (100)	98 (2)	100 (0)	99 (1)	96 (3)	39 (7)	55 (7)	76 (21)	78 (14)	75 (14)
Emile2k (200)	85 (3)	90 (2)	87 (2)	75 (12)	68 (4)	71 (6)	80 (0)	65 (4)	71 (3)
Emile2k (1000)	100 (0)	100 (0)	100 (0)	76 (7)	85 (8)	80 (6)	75 (3)	98 (3)	85 (3)
TA1 (200)	82 (7)	73 (5)	77 (5)	77 (3)	14 (3)	23 (4)	77 (24)	55 (12)	62 (14)
TA1 (2000)	95 (6)	100 (1)	97 (3)	98 (5)	48 (4)	64 (4)	50 (22)	92 (4)	62 (17)

Table 3.2 Experimental results. The training corpus sizes are indicated in the parentheses after the grammar names. P=Precision, R=Recall, F=F-score. The numbers in the table denote the performance estimates averaged over 50 trials, with the standard deviations in parentheses.

sentences were generated using either the learned grammar or the target grammar (as the case may be), and then parsed by the other grammar.

To ensure a fair comparison, we tuned the parameters of PCFG-BCL, EMILE and ADIOS on a separate dataset before running the evaluation experiments. Table 3.2 shows the experimental results. Each table cell shows the mean and standard deviation of performance estimates from 50 independent runs. In each run, each algorithm produced a single grammar as the output.

The results summarized in Table 3.2 show that PCFG-BCL outperformed both EMILE and ADIOS, on each of the test grammars, and by substantial margins on several of them. Moreover, in a majority of the tests, the standard deviations of the performance estimates of PCFG-BCL were lower than those of EMILE and ADIOS, suggesting that PCFG-BCL is more stable than the other two methods. It should be noted however, that neither EMILE nor ADIOS assume the training corpus to be generated from a PCFG, and thus they do not make full use of the distributional information in the training corpus. This might explain in part the superior performance of PCFG-BCL relative to EMILE and ADIOS.

We also examined the effect of grammar selection and grammar averaging (see Section 3.4.2), on the four datasets where PCFG-BCL did not achieve a perfect F-score on its own. In each case, we ran the algorithm for 10 times and then used the resulting grammars to perform grammar selection or grammar averaging as described in Section 3.4.2. The results (data not shown) show that grammar selection improved the F-score by 1.5% on average, and the largest increase of 4.4% was obtained on the TA1-200 data; grammar averaging improved the F-score

by 3.2% on average, and the largest increase of 9.3% was obtained also on the TA1-200 data. In addition, both grammar selection and averaging reduced the standard deviations of the performance estimates.

3.5.1 Experiments on Real World Data

We have also tested PCFG-BCL on a real-world natural language corpus, the Wall Street Journal corpus from the Penn Treebank, and evaluated the learned grammar on the basis of strong generative capacity (the PARSEVAL metric [Manning and Schütze (1999)]). The resulting score is worse than that of the right-branching baseline (i.e., assigning a right-branching parse tree to any sentence), even if the algorithm is enhanced with beam search. This bad performance is likely due to the combination of two factors: 1) the real-world natural language corpus is very sparse and noisy, so the statistics is very unreliable, which leads to erroneous grammar rule learning, and 2) the algorithm does not have the ability to reverse its rule learning and sentence parsing, so earlier errors can lead to more errors in later learning.

Note that other structure search approaches, like EMILE and ADIOS, have also been found to perform bad in learning real-world natural language grammars [Cramer (2007)]. This highlights a more fundamental limitation of structure search approaches: to solve the discrete optimization problem of structure search, they often resort to suboptimal methods that cannot scale up to real-world data that is sparse and noisy.

3.6 Conclusion and Discussion

3.6.1 Related Work

EMILE [Adriaans et al. (2000)] uses a simpler form of biclustering to create new nonterminals. It performs biclustering on an initial table constructed from the unreduced corpus, finding rules with only terminals on the right-hand side; and then it turns to the substitutability heuristic to find high-level rules. In contrast, PCFG-BCL performs iterative biclustering that finds both kinds of rules. ABL [van Zaanen (2000)] employs the substitutability heuristic to group possible constituents to nonterminals. Clark (2007) uses the "substitution-graph" heuristic or

distributional clustering [Clark (2001)] to induce new nonterminals and rules. These techniques could be less robust than the biclustering method, especially in the presence of ambiguity as discussed in Section 3.1 and also in [Adriaans et al. (2000)]. Both ABL and Clark's method rely on some heuristic criterion to filter non-constituents, whereas PCFG-BCL automatically identifies constituents as a byproduct of learning new rules from biclusters that maximize the posterior gain. ADIOS [Solan et al. (2005)] uses a probabilistic criterion to learn "patterns" (AND symbols) and the substitutability heuristic to learn "equivalence classes" (OR symbols). In comparison, our algorithm learns the two kinds of symbols simultaneously in a more unified manner.

The inside-outside algorithm [Baker (1979); Lari and Young (1990)], one of the earliest algorithms for learning PCFG, assumes a fixed, usually fully connected grammar structure and tries to maximize the likelihood, making it very likely to overfit the training corpus. Subsequent work has adopted the Bayesian framework to maximize the posterior of the learned grammar given the corpus [Chen (1995); Kurihara and Sato (2004)], and has incorporated grammar structure search [Chen (1995); Kurihara and Sato (2006)]. Our choice of prior over the set of candidate grammars is inspired by [Chen (1995)]. However, compared with the approach used in [Chen (1995)], PCFG-BCL adds more grammar rules at each step without sacrificing completeness (the ability to find any CFG); and the posterior re-estimation in PCFG-BCL is more straightforward and efficient (by using Eq.3.2 and 3.3).

3.6.2 Conclusion

We have presented PCFG-BCL, an unsupervised algorithm that learns a probabilistic context-free grammar (PCFG) from positive samples. The algorithm acquires rules of an unknown PCFG through iterative biclustering of bigrams in the training corpus. Results of our experiments on several synthetic benchmark datasets show that PCFG-BCL is competitive with the state-of-the-art structure search methods for learning CFG from positive samples.

CHAPTER 4. A Parameter Learning Approach with Unambiguity Regularization

The parameter learning approaches for learning probabilistic grammars assume a fixed set of grammar rules and try to learn their probabilities. Some parameter learning approaches, especially those encouraging parameter sparsity, can also be used to refine the set of grammar rules by removing rules with very small probabilities. The parameter learning approaches are typically more scalable than the structure search approaches, because parameter learning is a continuous optimization problem which is in general easier than the discrete optimization problem that the structure search approaches try to solve. Therefore, most of the state-of-the-art algorithms for unsupervised learning of natural language grammars belong to the parameter learning approaches.

In this chapter we introduce a novel parameter learning approach for learning natural language grammars based on the idea of *unambiguity regularization*. We first make the observation that natural language is remarkably unambiguous in the sense that each natural language sentence has a large number of possible parses but only a few of the parses are syntactically valid. We then incorporate this prior information of grammar unambiguity into parameter learning by means of posterior regularization [Ganchev et al. (2010)]. The resulting algorithm family contains classic EM and Viterbi EM, as well as a novel softmax-EM algorithm that can be implemented with a simple and efficient extension to classic EM. Our experiments show that unambiguity regularization improves natural language grammar learning, and when combined with other techniques our approach achieves the state-of-the-art grammar learning results.

4.1 Introduction

The simplest parameter learning approaches for unsupervised grammar learning optimize the likelihood of the grammar rule probabilities given the training data, typically by means of expectation-maximization (EM) [Baker (1979); Lari and Young (1990); Klein and Manning (2004)]. However, on real-world problems like natural language grammar learning, the training data is usually very sparse, so the maximum-likelihood grammar is very likely to overfit the training data. To avoid this problem, many of the more recent approaches incorporate prior information of the target grammar into learning. A Dirichlet prior over rule probabilities was used by Kurihara and Sato (2004) to smooth the probabilities, and was used by Johnson et al. (2007) (with less-than-one hyperparameters) to encourage sparsity of grammar rules. Finkel et al. (2007) and Liang et al. (2007) proposed the use of the hierarchical Dirichlet process prior which encourages a smaller grammar size without assuming a fixed number of nonterminals. Cohen et al. (2008) and Cohen and Smith (2009) employed the logistic normal prior to model the correlations between grammar symbols. Gillenwater et al. (2010) incorporated the structural sparsity bias into grammar learning by means of posterior regularization.

Recently, however, it was found that when Viterbi EM (also called hard EM) is used instead of classic EM, unsupervised learning of natural language grammars can be significantly improved even without using any prior information [Spitkovsky et al. (2010b)]. A similar observation was made in [Poon and Domingos (2011)] where a grammar-like model is learned from image data. This finding is somewhat surprising because Viterbi EM is a degeneration of classic EM and is therefore considered to be less effective in finding the optimum. Spitkovsky et al. (2010b) speculate that the observed advantage of Viterbi EM over classic EM is partly because classic EM reserves too much probability mass to spurious parses in the E-step, but it remains unclear why Viterbi EM is more likely to find the correct parses than classic EM.

In this chapter we propose to use a novel kind of prior information for natural language grammar learning, namely the syntactic unambiguity of natural language. It has been widely acknowledged that syntactic ambiguities (i.e., multiple syntactic parses exist for a single sentence) are ubiquitous in natural languages. However, the number of high-probability parses of

a typical natural language sentence is rather small, in comparison with the number of all possible parses which can be tremendous even for short sentences. In this sense, natural language grammars are impressively unambiguous. We incorporate this prior information into grammar learning by using the posterior regularization framework [Ganchev et al. (2010)]. The resulting algorithm family contains a novel softmax-EM algorithm which falls between classic EM and Viterbi EM. The softmax-EM algorithm can be implemented with a simple and efficient extension to classic EM. In addition, we show that Viterbi EM is a special case of our algorithm family, which gives an explanation of the good performance of Viterbi EM observed in previous work: it is because Viterbi EM implicitly utilizes unambiguity regularization. Our experiments on real-world natural language data show that unambiguity regularization improves natural language grammar learning, and when combined with other techniques our approach achieves the state-of-the-art grammar learning results.

The rest of this chapter is organized as follows. In section 4.2 we analyze the unambiguity of natural language grammars. In section 4.3 we formulate the unambiguity regularization for grammar learning and derive the algorithms. We show the experimental results in section 4.4 and conclude the chapter in section 4.5.

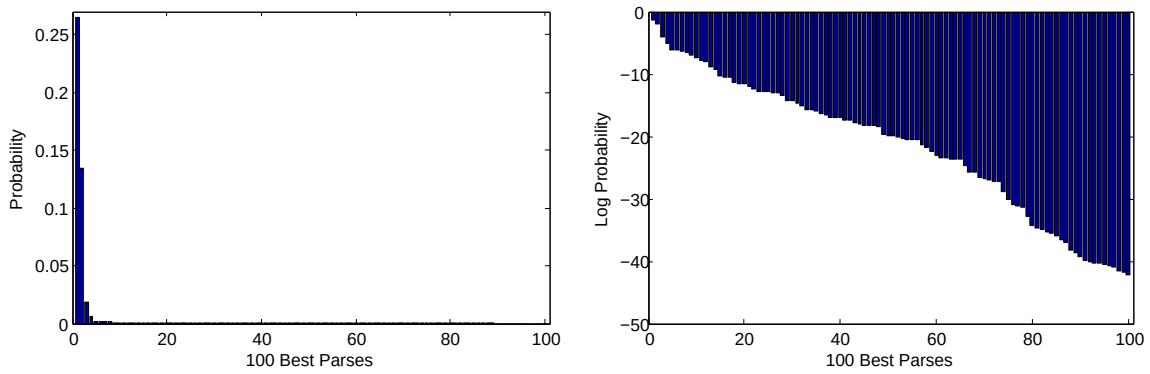
4.2 The (Un)ambiguity of Natural Language Grammars

A grammar is said to be ambiguous on a sentence if the sentence can be parsed in more than one way by the grammar. It is widely acknowledged that ambiguities are ubiquitous in natural languages, i.e., natural language grammars are ambiguous on a significant proportion of natural language sentences. For example, in [Manning and Schütze (1999), Chapter 12] the authors randomly choose a Wall Street Journal article and parse the first sentence:

> The post office will hold out discounts and service concessions as incentives.

They point out that even this randomly selected sentence has at least five plausible syntactic parses.

We parsed the same sentence using one of the stat-of-the-art English parser, the Berkeley parser [Petrov et al. (2006)]. In the parsing result we found the parses given in [Manning and

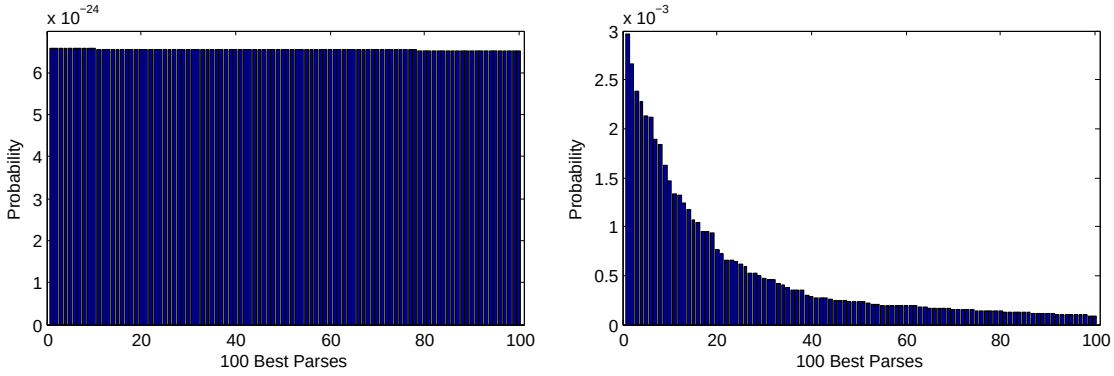


(a) (b)

Figure 4.1 The probabilities and log probabilities of the 100 best parses of the sample sentence.

Schütze (1999)] as well as many other alternative parses. Indeed, since the Berkeley parser uses a probabilistic context-free grammar, we estimate that the total number of possible parses[1] of the above sentence is 9×10^{40}. However, only a few of these parses have significant probabilities. Figure 4.1(a) shows the probabilities of the 100 parses of the sample sentence with the highest probabilities, and we can see that most of the parse probabilities are negligible compared to the probability of the best parse. Figure 4.1(b) shows the log probabilities of the same 100 best parses, and it can be seen that there is a roughly exponential decrease in the probabilities from the best parses to the less likely parses. We repeatedly parsed many other natural language sentences and observed very similar results. This observation suggests that natural language grammars are indeed remarkably unambiguous on natural language sentences, in the sense that for a typical natural language sentence, the probability mass of the parses is concentrated in a tiny portion of all possible parses (a few parses out of 9×10^{40} parses for our sample sentence). This is not surprising considering that the main purpose of natural language is communication and the evolutionary pressure for more efficient communication has certainly minimized the ambiguity in natural language over the tens of thousands of years since the origin of natural language.

[1] Given a sentence of length m and a Chomsky normal form grammar with n nonterminals, the number of all possible parses is $C_{m-1} \times n^{m-1}$, where C_{m-1} is the $(m-1)$-th Catalan number. This number is further increased if there are unary rules in the grammar.



(a) (b)

Figure 4.2 The probabilities of the 100 best parses of the sample sentence produced by a random grammar and a maximum-likelihood grammar learned by the EM algorithm.

To highlight the unambiguity of natural language grammars, here we compare the parse probabilities shown in Figure 4.1(a) with the parse probabilities produced by two other probabilistic context-free grammars. In figure 4.2(a) we show the probabilities of the 100 best parses of the sample sentence produced by a random grammar. The random grammar has a similar number of nonterminals as in the Berkeley parser, and its grammar rule probabilities are sampled from a uniform distribution and then normalized. It can be seen that unlike the natural language grammar, the random grammar produces a very uniform probability distribution. Figure 4.2(b) shows the probabilities of the 100 best parses of the sample sentence produced by a maximum-likelihood grammar learned from the unannotated Wall Street Journal corpus of the Penn Treebank using the EM algorithm. An exponential decrease can be observed in the probabilities, but the probability mass is still much less concentrated than in the case of the natural language grammar. Again, we repeated the experiments on other natural language sentences and observed similar results. This suggests that both the random grammar and the maximum-likelihood grammar are far more ambiguous on natural language sentences than the true natural language grammars.

4.3 Unambiguity Regularization

In order to learn a grammar that is unambiguous on natural language sentences, we need to incorporate this inductive bias into the learning process. To achieve this, we first need to formulate the ambiguity of a grammar on a sentence. Assume a grammar with a fixed set of grammar rules and let θ be the rule probabilities. Let x represent a sentence and let z represent its parse. Then one natural measurement of the ambiguity is the information entropy of z conditioned on x and θ:

$$H(z|x,\theta) = -\sum_{z} p_\theta(z|x) \log p_\theta(z|x)$$

The lower the entropy is, the less ambiguity there is in sentence x given the grammar. When the entropy reaches 0, the grammar is strictly unambiguous on sentence x, i.e., only one parse of the sentence has non-zero probability.

Now we need to modify the objective function of grammar learning to encourage low ambiguity of the learned grammar in parsing natural langauge sentences. One approach is to use a prior distribution that prefers a grammar with low ambiguity on the sentences it generates. Since the likelihood term in the objective function assures that the grammar has high probability to generate natural language sentences, combining the likelihood and the prior would encourage low ambiguity of the grammar on natural language sentences. Unfortunately, adding this prior distribution to the objective function leads to intractable inference. So here we adopt a different approach that uses the posterior regularization framework [Ganchev et al. (2010)]. Posterior regularization biases the learning towards the desired behavior by constraining the posterior probability on unlabeled data. In our case, we use the constraint that the posterior distributions on the parses of the training sentences must have low entropy, which is equivalent to requiring the learned grammar to have low ambiguity on the training sentences.

Let $\mathbf{X} = \{x_1, x_2, \ldots, x_n\}$ denote the set of training sentences, $\mathbf{Z} = \{z_1, z_2, \ldots, z_n\}$ denote the set of parses of the training sentences, and θ denote the rule probabilities of the grammar. We use the slack-penalized version of the posterior regularization objective function:

$$J(\theta) = \log p(\theta|\mathbf{X}) - \min_{q,\xi} \left(\mathbf{KL}(q(\mathbf{Z})||p_\theta(\mathbf{Z}|\mathbf{X})) + \sigma \sum_i \xi_i \right)$$

$$\text{s.t. } \forall i, H(z_i) = -\sum_{z_i} q(z_i) \log q(z_i) \leq \xi_i$$

where σ is a nonnegative constant that controls the strength of the regularization term; q is an auxiliary distribution such that $q(\mathbf{Z}) = \prod_i q(z_i)$. The first term in the objective function is the log posterior probability of the grammar parameters given the training corpus, and the second term minimizes the KL-divergence between the posterior distribution on $\mathbf{Z}$ and the auxiliary distribution q while constrains q to have low entropy. We can incorporate the constraint into the objective function, so we get

$$J(\theta) = \log p(\theta|\mathbf{X}) - \min_q \left(\mathbf{KL}(q(\mathbf{Z})||p_\theta(\mathbf{Z}|\mathbf{X})) + \sigma \sum_i H(z_i) \right)$$

To optimize this objective function, we can perform coordinate ascent on a two-variable function:

$$F(\theta, q) = \log p(\theta|\mathbf{X}) - \left(\mathbf{KL}(q(\mathbf{Z})||p_\theta(\mathbf{Z}|\mathbf{X})) + \sigma \sum_i H(z_i) \right) \tag{4.1}$$

We can also rewrite $F(\theta, q)$ as

$$\begin{aligned} F(\theta, q) &= \sum_{\mathbf{Z}} q(\mathbf{Z}) \log(p_\theta(\mathbf{X})p(\theta)) + \text{const} - \sum_{\mathbf{Z}} q(\mathbf{Z}) \log \frac{q(\mathbf{Z})}{p_\theta(\mathbf{Z}|\mathbf{X})} - \sigma \sum_i H(z_i) \\ &= \sum_{\mathbf{Z}} q(\mathbf{Z}) \log(p_\theta(\mathbf{X}, \mathbf{Z})p(\theta)) + \text{const} - \sum_{\mathbf{Z}} q(\mathbf{Z}) \log q(\mathbf{Z}) - \sigma \sum_i H(z_i) \end{aligned} \tag{4.2}$$

There are two steps in each coordinate ascent iteration. In the first step, we fix q and optimize θ (based on Eq.4.2).

$$\theta^* = \arg\max_\theta F(\theta, q) = \arg\max_\theta \mathbf{E}_q[\log(p_\theta(\mathbf{X}, \mathbf{Z})p(\theta))]$$

This is equivalent to the M-step in the EM algorithm. The second step fixes θ and optimizes q (based on Eq.4.1).

$$q^* = \arg\max_q F(\theta, q) = \arg\min_q \left(\mathbf{KL}(q(\mathbf{Z})||p_\theta(\mathbf{Z}|\mathbf{X})) + \sigma \sum_i H(z_i) \right)$$

It is different from the E-step of the EM algorithm in that it contains an additional regularization term $\sigma \sum_i H(z_i)$. Ganchev et al. (2010) propose to use the projected subgradient method to solve the optimization problem in this step in the general case of posterior regularization. In our case, however, an analytical solution can be derived as shown below.

First, note that the optimization objective of this step can be rewritten as sum over functions of individual training sentences.

$$\mathbf{KL}(q(\mathbf{Z})||p_\theta(\mathbf{Z}|\mathbf{X})) + \sigma \sum_i H(z_i) = \sum_i f_i(q)$$

where

$$\begin{aligned} f_i(q) &= \mathbf{KL}(q(z_i)||p_\theta(z_i|x_i)) + \sigma H(z_i) \\ &= \sum_{z_i} \left(q(z_i) \log \frac{q(z_i)}{p_\theta(z_i|x_i)} - \sigma q(z_i) \log q(z_i) \right) \\ &= \sum_{z_i} \left(q(z_i) \log \frac{q(z_i)^{1-\sigma}}{p_\theta(z_i|x_i)} \right) \end{aligned}$$

So we can optimize $f_i(q)$ for each training sentence x_i. The optimum of $f_i(q)$ depends on the value of the constant σ.

Case 1: $\sigma = 0$**.**

$f_i(q)$ contains only the KL-divergence term, so the second step in the coordinate ascent iteration becomes the standard E-step of the EM algorithm.

$$q^*(z_i) = p_\theta(z_i|x_i)$$

Case 2: $0 < \sigma < 1$**.**

The space of valid assignments of the distribution $q(z_i)$ is a unit m-simplex, where m is the total number of valid parses of sentence x_i. Denote this space by Δ. We have the following theorem, the proof of which can be found in Appendix B.

Theorem 4.1. *$f_i(q)$ is strictly convex on the unit simplex Δ.*

By applying the Lagrange multiplier, we get the stationary point of $f_i(q)$ on the unit simplex Δ:

$$q^*(z_i) = \alpha_i p_\theta(z_i|x_i)^{\frac{1}{1-\sigma}} \tag{4.3}$$

where α_i is the normalization factor

$$\alpha_i = \frac{1}{\sum_{z_i} p_\theta(z_i|x_i)^{\frac{1}{1-\sigma}}}$$

Because $f_i(q)$ is strictly convex on the unit simplex Δ, this stationary point is the global minimum.

Note that because $\frac{1}{1-\sigma} > 1$, $q^*(z_i)$ can be seen as the result of applying a version of softmax function on $p_\theta(z_i|x_i)$. To compute q^*, note that $p_\theta(z_i|x_i)$ is the product of a set of grammar rule probabilities, so we can raise all the rule probabilities of the grammar to the power of $\frac{1}{1-\sigma}$ and then run the normal E-step of the EM algorithm. The normalization of q^* is included in the normal E-step.

With q^*, the objective function becomes

$$\begin{aligned} F(\theta, q^*) &= \log p(\theta|\mathbf{X}) - \sum_i \sum_{z_i} \left(q(z_i) \log \frac{\left(\alpha_i p_\theta(z_i|x_i)^{\frac{1}{1-\sigma}}\right)^{1-\sigma}}{p_\theta(z_i|x_i)} \right) \\ &= \sum_i \left(\log p(x_i|\theta) - (1-\sigma)\log \alpha_i\right) + \log p(\theta) - \log p(\mathbf{X}) \\ &= \sum_i \left((1-\sigma) \log p(x_i|\theta)^{\frac{1}{1-\sigma}} + (1-\sigma) \log \sum_{z_i} p_\theta(z_i|x_i)^{\frac{1}{1-\sigma}} \right) + \log p(\theta) - \log p(\mathbf{X}) \\ &= (1-\sigma) \sum_i \log \sum_{z_i} p(z_i, x_i|\theta)^{\frac{1}{1-\sigma}} + \log p(\theta) - \log p(\mathbf{X}) \end{aligned}$$

The first term is proportional to the log "likelihood" of the corpus computed with the exponentiated rule probabilities. So we can use a parsing algorithm to efficiently compute the value of the objective function (on the training corpus or on a separate development corpus) to determine when the iterative coordinate ascent shall be terminated.

Case 3: $\sigma = 1$

We need to minimize

$$f_i(q) = -\sum_{z_i} \left(q(z_i) \log p_\theta(z_i|x_i)\right)$$

Because $\log p_\theta(z_i|x_i) \leq 0$ for any z_i, the minimum of $f_i(q)$ is reached at

$$q^*(z_i) = \begin{cases} 1 & \text{if } z_i = \arg\max_{z_i} p_\theta(z_i|x_i) \\ 0 & \text{otherwise} \end{cases}$$

Case 4: $\sigma > 1$

We have the following theorem. The proof can be found in Appendix B.

Theorem 4.2. *The minimum of $f_i(q)$ is attained at a vertex of the unit simplex Δ.*

Now we need to find out at which of the vertices of the unit simplex Δ the minimum of $f_i(q)$ is attained. At the vertex where the probability mass is concentrated at the assignment z, the value of $f_i(q)$ is $-\log p_\theta(z|x_i)$. So the minimum is attained at

$$q^*(z_i) = \begin{cases} 1 & \text{if } z_i = \arg\max_{z_i} p_\theta(z_i|x_i) \\ 0 & \text{otherwise} \end{cases}$$

It can be seen that the minimum in the case of $\sigma > 1$ is attained at the same point as in the case of $\sigma = 1$, where all the probability mass is assigned to the best parse of the training sentence. So q^* can be computed using the E-step of the Viterbi EM algorithm. Denote the best parse by z_i^*. With q^*, the objective function becomes

$$\begin{aligned} F(\theta, q^*) &= \log p(\theta|\mathbf{X}) + \sum_i \log p(z_i^*|x_i, \theta) \\ &= \sum_i (\log p(x_i|\theta) + \log p(z_i^*|x_i, \theta)) + \log p(\theta) - \log p(\mathbf{X}) \\ &= \sum_i \log p(z_i^*, x_i|\theta) + \log p(\theta) - \log p(\mathbf{X}) \end{aligned}$$

The first term is the sum of the log probabilities of the best parses of the corpus. So again we can use a parsing algorithm to efficiently compute it for the purpose of termination judgment.

Summary

Our learning algorithm with unambiguity regularization is an extension of the EM algorithm. The behavior of our algorithm is controlled by the value of the nonnegative constant σ. A larger value of σ induces a stronger bias towards an unambiguous grammar. When $\sigma = 0$, our algorithm is exactly the classic EM algorithm. When $\sigma \geq 1$, our algorithm is exactly the Viterbi EM algorithm, which considers only the best parses of the training sentences in the E-step. When $0 < \sigma < 1$, our algorithm falls between classic EM and Viterbi EM: it applies a softmax function (Eq.4.3) to the parsing distributions of the training sentences in the E-step. The softmax function can be computed by simply exponentiating the grammar rule probabilities at the beginning of the classic E-step, which does not increase the time complexity of the E-step. We call our algorithm in the case of $0 < \sigma < 1$ the *softmax-EM* algorithm.

4.3.1 Annealing the Strength of Regularization

In unsupervised probabilistic grammar learning, the initial grammar is typically very ambiguous (e.g., a random grammar or uniform grammar). So we need a value of the contant σ that is large enough to induce unambiguity in learning. On the other hand, natural language grammars do contain some degree of ambiguity, so if the value of σ is too large, then the induced grammar might be over-unambiguous and thus not a good model of natural languages. Therefore, it can be difficult to choose a proper value of σ.

One way to avoid choosing a fixed value of σ is to anneal its value. In the early stage of learning, the learner is typically in a "bad" region of the grammar space that contains highly ambiguous grammars, so we set a very large value of σ (e.g., $\sigma = 1$) to strongly push the learner towards the region of less ambiguous grammars. In the later stage of learning, we reduce the value of σ to avoid inducing too much unambiguity in the learned grammar. In fact, when the learner is already in a region of highly unambiguous grammars, it would be safe to reduce the value of σ to 0 (i.e., switching to classic EM).

4.3.2 Unambiguity Regularization with Mean-field Variational Inference

Variational inference approximates the posterior of the model given the training data. It typically leads to more accurate predictions than the MAP estimation. In addition, for certain types of prior distributions (e.g., a Dirichlet prior with less-than-one hyperparameters), the MAP estimation may not exist, while variational inference does not have this problem. Here we incorporate unambiguity regularization into mean-field variational inference.

The objective function with unambiguity regularization for mean-field variational inference is:

$$F(q(\theta), q(\mathbf{Z})) = \log p(\mathbf{X}) - \left(\mathbf{KL}(q(\theta)q(\mathbf{Z}) || p(\theta, \mathbf{Z}|\mathbf{X})) + \sigma \sum_i H(z_i) \right)$$
$$\text{where } \forall i, H(z_i) = -\sum_{z_i} q(z_i) \log q(z_i)$$

We can perform coordinate ascent that alternately optimizes $q(\theta)$ and $q(\mathbf{Z})$. Since the regularization term does not contain $q(\theta)$, the optimization of $q(\theta)$ is exactly the same as in the classic

mean-field variational inference. To optimize $q(\mathbf{Z})$, we have

$$q^*(\mathbf{Z}) = \arg\min_{q(\mathbf{Z})} \left(\mathbf{KL}(q(\mathbf{Z})||\widetilde{p}(\mathbf{X},\mathbf{Z})) + \sigma \sum_i H(z_i) \right)$$

where $\widetilde{p}(\mathbf{X},\mathbf{Z})$ is defined as

$$\log \widetilde{p}(\mathbf{X},\mathbf{Z}) = E_{q(\theta)}[\log p(\theta,\mathbf{Z},\mathbf{X})] + \text{const}$$

Now we can follow the same derivation as in the MAP estimation with unambiguity regularization, and the result is also the same except that $p_\theta(z_i|x_i)$ is replaced with $\widetilde{p}(x_i,z_i)$ in the four cases.

Note that if Dirichlet priors are used over grammar rule probabilities θ, then in mean-field variational inference $\widetilde{p}(x_i,z_i)$ can be represented as the product of a set of weights Kurihara and Sato (2004). Therefore in order to compute $q^*(z_i)$, in the case of $0 < \sigma < 1$, we simply need to raise all the weights to the power of $\frac{1}{1-\sigma}$ before running the normal step of computing $q^*(z_i)$ in classic mean-field variational inference; and in the case of $\sigma \geq 1$, we can simply use the weights to find the best parse of the training sentence and assign probability 1 to it.

4.4 Experiments

We tested the effectiveness of unambiguity regularization in unsupervised learning of a type of dependency grammar called the dependency model with valence (DMV) [Klein and Manning (2004)]. We used section 2-21 of the Wall Street Journal corpus of the Penn Treebank for training, and section 23 of the same corpus for testing. We trained the learner on the unannotated sentences of length ≤ 10 with punctuation stripped in the training corpus. We started our algorithm with the informed initialization proposed in [Klein and Manning (2004)], and terminated the algorithm when the increase in the value of the objective function fell below a threshold of 0.01%. To evaluate the learned grammars, we used each grammar to parse the testing corpus and the resulting dependency parses were compared against the gold standard parses. The percentage of the dependencies that were correctly matched was output as the dependency accuracy (DA) of the grammar. We report the dependency accuracy on sentences of length ≤ 10, ≤ 20 and all sentences.

σ	Testing Accuracy		
	≤ 10	≤ 20	All
0 (classic EM)	46.6	40.3	35.5
0.25	53.9	44.8	**40.3**
0.5	52.0	42.9	38.8
0.75	51.5	43.1	38.8
1 (Viterbi EM)	**58.3**	**45.4**	39.5

Table 4.1 The dependency accuracies of grammars learned by our algorithm with different values of σ.

4.4.1 Results with Different Values of σ

We compared the performance of our algorithm with five different values of the constant σ: 0 (i.e., classic EM), 0.25, 0.5, 0.75, 1 (i.e., Viterbi EM). Table 4.1 shows the experimental results. It can be seen that learning with unambiguity regularization (i.e., with $\sigma > 0$) consistently outperforms learning without unambiguity regularization (i.e., $\sigma = 0$). The grammar learned by Viterbi EM has significantly higher dependency accuracy in parsing short sentences. We speculate that this is because short sentences are less ambiguous and therefore a strong unambiguity regularization is especially helpful in learning the grammatical structures of short sentences. In parsing sentences of all lengths, $\sigma = 0.25$ achieves the best dependency accuracy, which suggests that explicitly controlling the strength of unambiguity regularization can improve learning.

4.4.2 Results with Annealing and Prior

We annealed the value of σ from 1 to 0 when running our algorithm. Since it takes about 30 iterations for classic EM to converge on the training set, we reduce the value of σ at a constant speed such that it reaches 0 at iteration 30. We also tested other choices of the annealing speed and found that the algorithm is insensitive to it. The first row in Table 4.2 shows the experimental result. It can be seen that annealing the value of σ not only circumvents the problem of choosing a proper value of σ, but also significantly improves the learning result than using any fixed value of σ.

Dirichlet priors with less-than-one hyperparameters are often used to induce parameter sparsity. We added Dirichlet priors over grammar rule probabilities and ran the variational

	Testing Accuracy		
	≤ 10	≤ 20	All
UR with Annealing	62.9	52.4	47.3
UR with Annealing&Prior	**64.2**	**55.5**	**50.2**
PR-S	62.1	53.8	49.1
LN Families	59.3	45.1	39.0
SLN TieV&N	61.3	47.4	41.4

Table 4.2 The dependency accuracies of grammars learned by our algorithm (denoted by "UR") with annealing and prior, compared with previous published results.

inference version of our algorithm. We experimented with different values of the hyperparameter α and found that 0.25 is the best value, which is consistent with the result from previous work [Cohen et al. (2008); Gillenwater et al. (2010)]. When tested with different values of σ, adding Dirichlet priors with $\alpha = 0.25$ consistently boosted the dependency accuracy of the learned grammar by 1–2. When the value of σ was annealed during variational inference with Dirichlet priors, the dependency accuracy was further improved, as shown in the second row of Table 4.2.

Table 4.2 also compares our results with the best results that have been published in the literature for unsupervised learning of DMV with the same training set. PR-S is a posterior regularization approach that encourages sparsity in dependency types [Gillenwater et al. (2010)]. LN Families is learning with logistic normal priors with manual initialization of the covariance matrices [Cohen et al. (2008)]. SLN TieV&N is a version of learning with shared logistic normal priors [Cohen and Smith (2009)]. It can be seen that our best result (unambiguity regularization with annealing and prior) clearly outperforms previous results. In addition, we expect our algorithm to be more efficient than the other approaches, because our algorithm only inserts an additional parameter exponentiation step into each iteration of classic EM or variational inference, while all the other approaches involve additional optimization steps (using gradient descent) in each iteration.

4.5 Conclusion and Discussion

4.5.1 Related Work

Our work is motivated by the discovery of the advantage of Viterbi EM over classic EM in learning grammars from natural language data [Spitkovsky et al. (2010b)] and image data [Poon and Domingos (2011)]. As we have shown in this chapter, Viterbi EM implicitly employs the unambiguity regularization in learning, which gives an explanation of its good performance.

The sparsity bias, which encourages sparsity of grammar rules, has been widely used in unsupervised grammar learning (e.g., Chen (1995); Johnson et al. (2007); Gillenwater et al. (2010)). The sparsity bias is related to the unambiguity bias in the sense that a sparser grammar is in general less ambiguous. However, sparsity and unambiguity are conceptually different and lead to different mathematical formulations. Also, for natural language grammars, their sparsity and unambiguity might be the results of different evolutionary pressures.

4.5.2 Conclusion and Future Work

We have introduced a novel inductive bias for learning natural language grammars called unambiguity regularization. It is based on the fact that natural language grammars are remarkably unambiguous, i.e., in parsing natural language sentences they tend to concentrate the probability mass to a very small portion of all possible parses. We incorporate this inductive bias into learning by using posterior regularization. The resulting algorithm family contains classic EM and Viterbi EM, as well as a novel softmax-EM algorithm which falls between classic EM and Viterbi EM. The softmax-EM algorithm can be implemented with a simple and efficient extension to classic EM. Our experiments on real-world natural language data show that unambiguity regularization improves natural language grammar learning, and by incorporating regularization strength annealing and sparsity priors our approach achieves the state-of-the-art grammar learning result.

For future work, it is interesting to try other formulations of unambiguity regularization. One may try measurements of ambiguity other than the entropy. One may also try applying a nonlinear loss function to the ambiguity of the grammar in order to allow some degree of

ambiguity as observed in natural language.

CHAPTER 5. An Incremental Learning Approach by Using Curricula

The incremental learning approaches for learning probabilistic grammars are meta-algorithms that specify a series of intermediate learning targets which culminate in the actual learning target. These meta-algorithms can utilize either structure search approaches or parameter learning approaches as the subroutine. For some very complicated real-world grammars (e.g., natural language grammars), incremental approaches can provide a better learning result than the direct application of structure search or parameter learning approaches.

In this chapter, we study a particular type of incremental learning, namely learning with a curriculum (a means of presenting training samples in a meaningful order). We introduce the *incremental construction hypothesis* that explains the benefits of a curriculum in learning grammars and offers some useful insights into the design of curricula as well as learning algorithms. We present results of experiments with (a) carefully crafted synthetic data that provide support for our hypothesis and (b) natural language corpus that demonstrate the utility of curricula in unsupervised learning of probabilistic grammars.

5.1 Introduction

Much of the existing work on unsupervised grammar learning (e.g., [Lari and Young (1990); Klein and Manning (2004); Cohen et al. (2008)]) starts with all the sentences of a training corpus and tries to learn the whole grammar. In contrast, there is a substantial body of evidence that humans and animals learn much better when the data are not randomly presented but organized into a *curriculum* that helps expose the learner to progressively more complex concepts or grammatical structures. Such a learning strategy has been termed *curriculum learning* by Bengio et al. (2009). There has been some effect to apply curriculum learning to unsupervised

grammar learning. The results of a seminal experimental study by Elman (1993) suggested that grammar induction using recurrent neural networks can benefit from *starting small*, i.e., starting with restrictions on the data or on the capacity of the learner, and gradually relaxing the restrictions. However, the experiments of Rohde and Plaut (1999) called into question the benefits of *starting small* in language acquisition. A more recent study by Spitkovsky et al. (2010a) offered evidence that is suggestive of the benefits of curricula in probabilistic grammar induction. To explain the benefits of curricula, Bengio et al. (2009) hypothesized that a well-designed curriculum corresponds to learning starting with a smoothed objective function and gradually reducing the degree of smoothing over successive stages of the curriculum, thus guiding the learning to better local minima of a non-convex objective function. The precise conditions on the curriculum or the learner that lead to improved learning outcomes are far from well-understood.

Against this background, we explore an alternative explanation of the benefits of curricula, especially in the context of unsupervised learning of probabilistic grammars. Our explanation is based on the *incremental construction hypothesis* (ICH) which asserts that when the target of learning is a structure (in our case, a probabilistic grammar) that can be decomposed into a set of sub-structures (in our case, grammar rules), an ideal curriculum gradually emphasizes data samples that help the learner to successively discover new sub-structures. This hypothesis, if true, can help guide the design of curricula as well as learning algorithms. We present results of experiments on synthetic data that provide support for ICH; and we demonstrate the utility of curricula in unsupervised learning of grammars from a real-world natural language corpus.

5.2 Curriculum Learning

As noted by Bengio et al. (2009), at an abstract level a curriculum can be seen as a sequence of training criteria. Each training criterion in the sequence is associated with a different set of weights on the training samples, or more generally, with a re-weighting of the training distribution. Thus, we can model a curriculum as a sequence of weighting schemes $\langle W_1, W_2, \ldots, W_n \rangle$. The first weighting scheme W_1 assigns larger weights to "easier" samples, and each subsequent weighting scheme increases the weights assigned to "harder" samples, until the last weighting

scheme W_n that assigns uniform weights to the training samples. The measure of "hardness" of training samples depends on the learning problem and learning algorithm. Ideally, the information entropy of the weighting schemes increases monotonically, i.e., $\forall i < j, H(W_i) < H(W_j)$. Given a curriculum, learning proceeds in an iterative fashion: at iteration i, the learner is initialized with the model f_{i-1} learned from the previous iteration, and is provided with the training data weighted by the weighting scheme W_i, based on which it generates a new model f_i. The final output of curriculum learning is f_n, the model produced by the last (n-th) iteration.

The baby-step algorithm [Spitkovsky et al. (2010a)] for unsupervised grammar learning can be seen as an instance of learning with a curriculum. The training data consist of a set of unannotated sentences. The hardness of a sentence is measured by its length (number of words). The i-th weighting scheme W_i assigns a weight of one to each sentence that consists of no more than i words and a weight of zero to any of the other sentences (thus specifying a subset of training sentences). At iteration i of learning, the expectation-maximization algorithm [Lari and Young (1990)] is run to convergence on the subset of training data specified by W_i, and the resulting grammar G_i is then used to initialize iteration $i + 1$. This curriculum introduces increasingly longer sentences into the training data seen by the learner, with the entire training corpus being provided to the learner at the last iteration, which produces the final output grammar.

5.3 The Incremental Construction Hypothesis of Curriculum Learning

We explore the incremental construction hypothesis (ICH) as a possible explanation of curriculum learning, in the context of learning probabilistic grammars. The hypothesis asserts that an ideal curriculum gradually emphasizes data samples that help the learner to successively discover new sub-structures (i.e., grammar rules) of the target grammar, which facilitates the learning. Formally, we define an ideal curriculum for grammar learning suggested in ICH as follows.

Definition 5.1. *A curriculum $\langle W_1, W_2, \ldots, W_n \rangle$ for learning a probabilistic grammar G of a pre-specified class of grammars C is said to satisfy incremental construction if the following*

three conditions are met.

1. *for any weighting scheme W_i, the weighted training data corresponds to a sentence distribution defined by a probabilistic grammar $G_i \in C$;*

2. *if R_i and R_j denote the sets of rules of the probabilistic grammars G_i and G_j respectively, then for any i, j s.t. $1 \leq i < j \leq n$, we have $R_i \subseteq R_j$;*

3. *for any i, j s.t. $1 \leq i, j \leq n$, and for any two grammar rules r_1, r_2 with the same rule condition (left-hand side) that appear in both G_i and G_j, we have*

$$\frac{P(r_1|G_i)}{P(r_2|G_i)} = \frac{P(r_1|G_j)}{P(r_2|G_j)}$$

In order words, an ideal curriculum that satisfies incremental construction specifies a sequence of intermediate target grammars $\langle G_1, G_2, \ldots, G_n \rangle$, and each intermediate grammar G_i is a sub-grammar of the next intermediate grammar G_{i+1}. Note that curriculum learning requires the last weighting scheme W_n to be uniform, so given enough training data, the last grammar G_n in the sequence should be weakly equivalent to the target grammar G, i.e., they define the same distribution of sentences.

The third of the three conditions in Definition 5.1 implies that for a grammar rule that appears in two consecutive grammars, its probability either remains unchanged or, if one or more new rules that share the same left-hand side of the rule are introduced in the second grammar, is renormalized to a smaller value that preserves the probability ratios of this rule to other rules that share the same left-hand side. However, since the training data is usually sparse and sometimes noisy in practice, it would be almost impossible to find a curriculum that exactly satisfies the third condition. Therefore we can relax this condition as follows.

3b. *for any i, j s.t. $1 \leq i < j \leq n$, and for any grammar rule r that appears in both G_i and G_j, we have $P(r|G_i) \geq P(r|G_j)$*

In order to be able to meaningfully assess the benefits of curricula in grammar learning, we need some measures of distance between two probabilistic grammars. There are two commonly used measures. The first is the distance between the parameter vectors (i.e., the vectors of rule

probabilities) of the two grammars. For each rule condition p in grammar G_i, the probabilities of the grammar rules with condition p constitute a multinomial vector (in the case that G_i contains no such rule, we add a dummy rule $p \rightarrow \varepsilon$ with probability 1). Let the parameter vector θ_i of a grammar G_i be the concatenation of the multinomial vectors of all the rule conditions. To make the parameter vectors of different grammars comparable, the elements of different parameter vectors are aligned such that a given rule occupies the same position in the parameter vector of each of the grammars $G_1 \ldots G_n$. The second distance measure is the distance between the distributions of grammatical structures (parses) defined by the two grammars. We can use the *total variation distance* of two distributions (defined as one half of the L_1 distance between them) for this purpose.

Now we can express the advantages of an ICH-based ideal curriculum (Definition 5.1) in the form of the following theorem.

Theorem 5.1. *If a curriculum $\langle W_1, W_2, \ldots, W_n \rangle$ satisfies incremental construction (with either condition 3 or 3b), then for any i, j, k s.t. $1 \leq i < j < k \leq n$, we have*

$$\begin{aligned} d_1(\theta_i, \theta_k) &\geq d_1(\theta_j, \theta_k) \\ d_{TV}(G_i, G_k) &\geq d_{TV}(G_j, G_k) \end{aligned}$$

where $d_1(\cdot, \cdot)$ denotes the L_1 distance; $d_{TV}(G_i, G_j)$ represents the total variation distance between the two distributions of grammatical structures defined by G_i and G_j.

The proof of the theorem exploits the fact that both the L_1 norm of the parameter vector and the sum of probabilities over all grammatical structures are constant regardless of the values of i, j and k. We give the detailed proof in Appendix C. This theorem shows that for any $i < j < k$, G_j is a better approximation of G_k than G_i. Therefore, it follows that each stage of curriculum learning tries to induce a grammar that provides a better initialization for the next stage of learning than any of the previous grammars, and the sequence of grammars $\langle G_1, G_2, \ldots, G_n \rangle$ offers a guided sequence of intermediate learning targets culminating in G_n.

In the case of some curricula that have been used in practice (e.g., the length-based curriculum in [Spitkovsky et al. (2010a)]), condition 3b appears to be still too strong. As will

be shown in Section 5.5, a curriculum may gradually introduce a new grammar rule to the learner across multiple stages. In this case, the probability of the new rule in the sequence of intermediate target grammars does not instantly jump from 0 to its actual value, but instead increases from 0 to its actual value through a series of small changes over several stages. We can prove a theorem similar to Theorem 5.1 in this setting:

Theorem 5.2. *If a curriculum $\langle W_1, W_2, \ldots, W_n \rangle$ satisfies the first two conditions in Definition 5.1 as well as a further relaxed version of the third condition:*

3c. for any grammar rules r, $P(r|G_i)$ first monotonically increases with i and then monotonically decreases with i.

then for any i, j, k s.t. $1 \leq i < j < k \leq n$, we have

$$d_1(\theta_i, \theta_k) \geq d_1(\theta_j, \theta_k)$$

The proof is similar to that of Theorem 5.1 and is given in Appendix C. However, under condition 3c, the second inequality for the total variation distance of grammars in Theorem 5.1 no longer holds.

5.3.1 Guidelines for Curriculum Design and Algorithm Design

ICH offers some guidance on how to design effective curricula. First, an effective curriculum should approximately satisfy the three conditions discussed above. Second, it should effectively break down the target grammar to be learned into as many chunks as possible, so that at each stage of learning the set of new rules introduced by the curriculum can be small and hence easy to learn. Quantitatively, this makes the distance between any two consecutive grammars G_i and G_{i+1} in the sequence $G_1 \ldots G_n$ as small as possible. Third, at each iteration an effective curriculum should introduce the new rule that results in the largest number of new sentences being added into the training data seen by the learner. This ensures that the learner has as many training sentences as possible for learning the new rule. From a theoretical perspective, since each new rule introduced into a grammar leads to some new grammatical structures that were previously invalid (i.e., had zero probabilities in the absence of the new rule), ideally at

each iteration the curriculum should introduce the rule that leads to a set of new grammatical structures with the highest sum of probabilities. The third guideline entails two special cases. First, if there are dependencies between rules (i.e., one rule is required for the other rule to be used), then the curriculum should conform to the the partial order defined by the dependencies. Second, among rules that share the same left-hand side, the curriculum should introduce rules in the descending order of their probabilities in the target grammar.

ICH also offers some guidance on designing learning algorithms. Because the learning target at each stage of the curriculum is a partial grammar, it is especially important for the learning algorithm to avoid the over-fitting to this partial grammar that hinders the acquisition of new grammar rules in later stages. Indeed, from our experiments (see the next two sections), we find that if adequate care is not exercised to minimize over-fitting, the results of learning with a curriculum can be worse than the results of learning without curriculum.

5.4 Experiments on Synthetic Data

To explore the validity of ICH, we designed a set of experiments using synthetic data generated from a known target grammar. With the target grammar known, we were able to construct the ideal curricula suggested by ICH. The grammar formalism that we used is the dependency model with valence (DMV) ([Klein and Manning (2004)], see Chapter 2), which has been shown to be amenable to unsupervised learning. We used the dependency treebank grammar of WSJ30 (the set of sentences no longer than 30 in the Wall Street Journal corpus of the Penn Treebank) as our target grammar, and generated a corpus of 500 sentences using this grammar. Expectation-maximization (EM) was used as the base learning algorithm. To deal with the problem of over-fitting mentioned in Section 5.3.1, we used a dynamic smoothing factor that is set to a large value initially when the effective training set seen by the learner is small; and is decreased as the learner is exposed to more training data. Five-fold cross-validation was used for evaluation: each time 100 sentences were used for training and the rest were used for evaluation. The results reported correspond to averages over the 5 cross-validation runs. Since we knew the correct parses of all the sentences, we used the standard PARSEVAL measures [Manning and Schütze (1999)] to evaluate the learned grammars.

We compared the performance of the learning algorithm when trained with seven different types of curricula as well as without a curriculum. In each curriculum, we used weights of either zero or one in the weighting schemes, which is tantamount to selecting a subset of the training corpus at each stage of the curriculum.

Ideal Curricula that satisfy all the ICH-based guidelines of curriculum design. We construct a curriculum as follows. Given the target grammar and the training set, at each stage of the curriculum we add to the partial grammar the smallest number of new rules of the target grammar that lead to the largest number of new sentences being added to the training set seen by the learner. We assign weight one to each of the training sentences that can be generated by the partial grammar. When there is a tie between two sets of new rules, we randomly select one.

Sub-Ideal Curricula that satisfy the first two guidelines of curriculum design. At each stage, we randomly add a new rule to the partial grammar and assign weight one to each of the sentences in the training corpus that can be generated by the partial grammar.

Random Curricula that add new training sentences at random to the training set at each stage of the curricula. We set the number of stages to be the same as that of IDEAL CURRICULA to ensure a fair comparison.

Ideal10, Sub-Ideal10 and Random10 curricula that are variants of IDEAL, SUB-IDEAL and RANDOM curricula respectively except that each stage in the curricula introduces at least 10 new training sentences. Therefore these curricula contain fewer stages.

Length-based Curriculum that introduces new training sentences ordered by their lengths, such that the learner is exposed to shorter sentences before it encounters longer sentences, as described in Section 5.2.

Figure 5.1 shows the mean PARSEVAL F-score from cross-validation for each type of curricula as well as learning without curriculum (labeled as PLAINEM). The construction procedure of the first six types of curricula is nondeterministic, so we present the mean F-score and standard deviation obtained from experiments with ten curricula of each type.

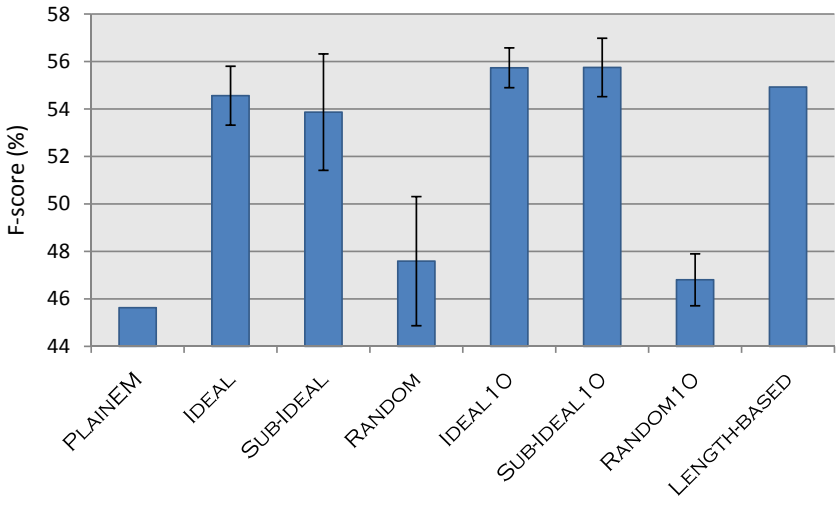


Figure 5.1 Comparison of the PARSEVAL F-scores of plain EM and learning with seven types of curricula. For each of the six types of curricula that involve nondeterministic construction, ten different curricula were constructed and tested and the mean F-score and standard deviation is shown.

The results of these experiments show that learning with any of the seven types of curricula, including the random ones, leads to better performance than learning without a curriculum. A possible explanation for the observed gains from the two types of random curricula could be that the target grammar used in this experiment tends to use a rather different set of rules to generate each sentence in the corpus, which would imply that with a small training corpus like ours, even a random partition of the sentences is likely to yield a curriculum that satisfies incremental construction to some extent. The results obtained using the four types of ideal and sub-ideal curricula are significantly better than those obtained using the random curricula. This is consistent with ICH (i.e., the first guideline of curriculum design). Each of the two types of ideal curricula has a slightly better mean F-score and a smaller standard deviation than the corresponding sub-ideal curricula, which suggests that the third guideline of curriculum design also helps facilitate learning. However, to the contrary of the second guideline, Ideal and Sub-Ideal have slightly worse performance than Ideal10 and Sub-Ideal10. We speculate that it is because curricula with more stages are more prone to the over-fitting problem discussed in Section 5.3.1.

Interestingly, Length-based Curriculum shows performance that is comparable to the

	Length-based vs Ideal	Sub-Ideal vs Ideal	Random vs Ideal
Kendall	0.7641	0.4125	0.0306
Spearman	0.9055	0.5672	0.0442

Table 5.1 Average correlations of three types of curricula with the Ideal curricula. Two types of rank correlation, Kendall's and Spearman's correlation, are shown.

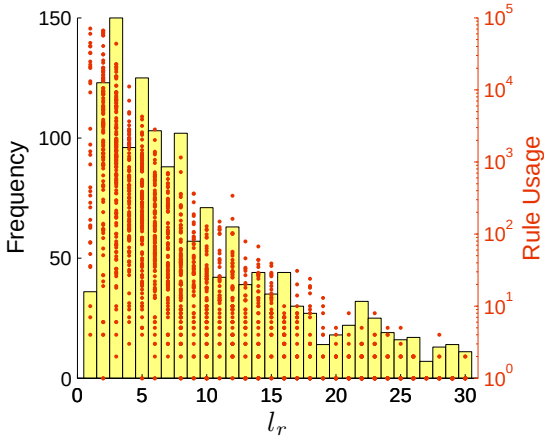
four types of ideal and sub-ideal curricula. To explore why this might be the case, we measured how similar the Length-based curriculum is to the Ideal curricula. Since in this set of experiments, each curriculum corresponds to an ordering of the sentences in the training corpus, we can compute the correlation between the orderings to measure the similarity of different curricula. We used two types of rank correlation, Kendall's correlation and Spearman's correlation, for this purpose. Table 5.1 shows the correlation between Length-based and Ideal, along with the correlations of Sub-Ideal and Random with Ideal for comparison. Because our experiments used ten different Ideal, Sub-Ideal and Random curricula, we report the average values of the correlations between curricula of different types. It can be seen that the Length-based curriculum is very similar to the Ideal curricula in the case of the training corpus and target grammar used in this experiment.

5.5 Experiments on Real Data

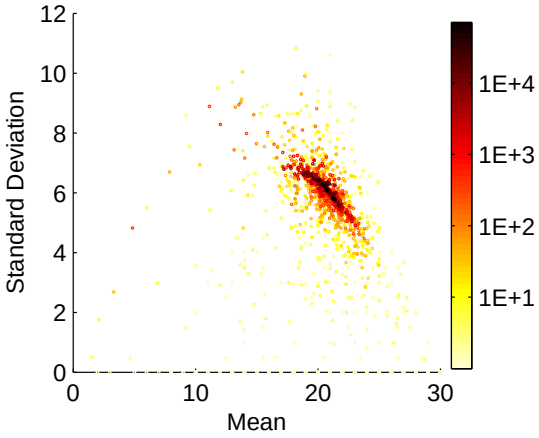
5.5.1 Analysis of Length-based Curriculum

In practice, since little is known about the target grammar when doing unsupervised learning, it is very difficult, if not impossible, to construct an ideal curriculum suggested by ICH. Hence, curricula that can be constructed without knowledge of the target grammar are preferred. The length-based curriculum offers an example of such curricula. In Section 5.4, we have shown that on the synthetic data generated from a real-world treebank grammar, the length-based curriculum is a good approximation of an ideal curriculum. In this subsection, we offer some evidence that this may still be true in the case of a real-world natural language corpus.

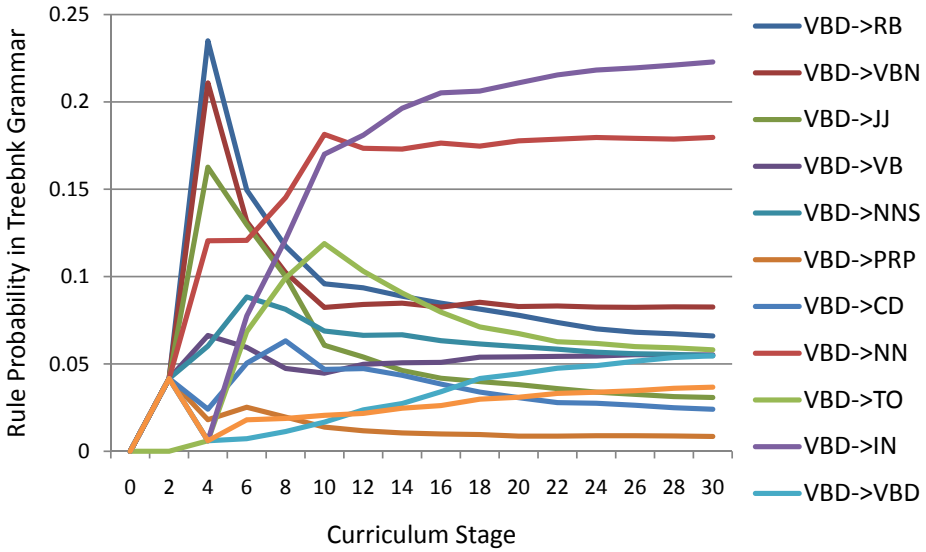
We use the WSJ30 corpus (the set of sentences no longer than 30 in the Wall Street Journal



(a) The bar graph shows the histogram of l_r (the length of the shortest sentence in the set of sentences that use rule r). Each point in the overlay corresponds a grammar rule r (with x-coordinate being l_r and y-coordinate being the number of times rule r is used in the whole corpus).



(b) Each point in the plot corresponds to a grammar rule r, with its x-coordinate being the mean length of sentences in S_r (the set of sentences in which rule r is used), y-coordinate being the corresponding standard deviation, and color indicating the number of times r is used in the whole corpus (with hotter colors denoting greater frequency of usage).



(c) The change of probabilities of VBD-headed rules with the stages of the length-based curriculum in the treebank grammars (best viewed in color). Rules with probabilities always below 0.025 are omitted.

Figure 5.2 Analysis of the length-based curriculum in WSJ30

corpus of the Penn Treebank) to learn a DMV grammar. Since we know the correct parse of each sentence in WSJ30, we can find the grammar rules that are used in generating each sentence. For a grammar rule r, let S_r be the set of sentences in which r is used, and let l_r be the length of the shortest sentence in S_r. Some statistics of grammar rule usage in WSJ30 are shown in Figure 5.2(a) and 5.2(b). The histogram in Figure 5.2(a) in fact shows the distribution of the stages at which the grammar rules are introduced in the length-based curriculum. It can be seen that the introduction of grammar rules is spread throughout the entire curriculum, as required by ICH (although more rules are introduced in the early stages). From the overlay plot in Figure 5.2(a) we can also see that rules that are used more frequently tend to be introduced earlier in the curriculum, which is consistent with the third guideline of curriculum design in Section 5.3.1. In Figure 5.2(b), most rules fall within a continuum that ranges from intermediate mean and high standard deviation to high mean and low standard deviation. This suggests that for any grammar rule r, in most cases, the lengths of the sentences in S_r distribute relatively evenly in the interval of $[l_r, 30]$ (where 30 is the length of the longest sentence in WSJ30). So in the length-based curriculum, rules learned in earlier stages can help parse the sentences introduced in later stages of the curriculum, thus facilitating the acquisition of new rules in later stages. This is also consistent with the third guideline of curriculum design.

With the correct parses being known for all the sentences in WSJ30, we can further construct the treebank grammar, in which the rule probabilities are computed from the number of times each rule is used in the parsed corpus. Since each stage of the length-based curriculum specifies a subset of the training sentences, we can construct a sequence of such treebank grammars, one for each stage in the curriculum. Each such grammar is the maximal likelihood grammar of the correct parses of the corresponding sub-corpus, so we can assume that condition 1 in Definition 5.1 is satisfied. Since each stage of the length-based curriculum adds new sentences to the sub-corpus that is available to the learner, it is easy to see that in this sequence of treebank grammars, once a rule is learned its probability can never drop to zero. This ensures that condition 2 in Definition 5.1 is also satisfied. How about condition 3? Figure 5.2(c) shows, for grammar rules that are conditioned on the VBD (past tense verb) head and the right dependency, how the rule probabilities change over the sequence of treebank grammars.

We note that most rule probabilities shown in the figure first increase over multiple stages (implying that the rules are being gradually introduced), and then monotonically decrease (due to renormalization of the probabilities as other rules are being introduced). We find that other grammar rules also behave similarly in relation to the sequence of treebank grammars. Therefore, the original condition 3 in Definition 5.1 is clearly violated, but its relaxed version, condition 3c in Theorem 5.2, is approximately satisfied. Therefore, the theoretical guarantee of Theorem 5.2 is likely to hold for the length-based curriculum for the WSJ30 corpus.

Furthermore, from Figure 5.2(c) we can see that rules are introduced in a specific order. Among the first rules to be introduced are those that produce RB, VBN, JJ and VB (as adverbials, predicatives, etc.); followed by rules that produce NNS, PRP and CD (as objects, etc.); followed by rules that produce NN (as objects) and TO (to head preposition phrases); and ending with rules that produce IN, VBD and VBG (for preposition phrases and clauses). This confirms that rules are introduced incrementally in the length-based curriculum.

5.5.2 Learning Results

We tested curriculum learning of DMV grammars from the unannotated WSJ30 corpus. Following the standard procedure for evaluating natural language parsers, section 2-21 of WSJ30 were used for training, section 22 was used for development, and section 23 was used for testing. We used expectation-maximization (EM) as the base learning algorithm, with an initialization of the grammar as described in [Klein and Manning (2004)]. To minimize the over-fitting problem discussed in Section 5.3.1, at each stage of the curriculum we terminated training when the likelihood of the development set stopped increasing. In addition, we set the maximal number of iterations at each stage (except the last stage) of the curriculum to a relatively small value, which further alleviates over-fitting while also speeding up the algorithm.

In addition to plain EM and the length-based curriculum, we tested a novel curriculum based on the likelihood of sentences. Because the use of EM as the base learning algorithm guarantees that at any time of the learning we have a complete grammar, we can use the negative log likelihood of a sentence given this grammar as a measure of the relative hardness of the sentence. With this likelihood-based hardness measure, we can construct a new curriculum

similar to the length-based curriculum, i.e., sentences with higher likelihood receive larger weights at earlier stages in the curriculum. However, because the grammar used to estimate the hardness of a sentence is continuously updated as a result of learning, so is the hardness measure, making the resulting curriculum an "active" curriculum. We repeated the analysis described in Section 5.5.1 on this new curriculum, and found the results similar to those reported for the length-based curriculum (data not shown).

In the curricula discussed in Section 5.4, the weights are set to either zero or one in the weighting schemes, and the set of sentences with weight one expands over successive stages of the curriculum. Here we also tested a different method: a continuous-valued weighting function is used to assign greater weights to easier sentences and less weights to harder sentences, and the weighting function becomes increasingly uniform over successive stages of the curriculum.

We evaluated all the intermediate grammars produced in the course of learning as well as the grammars that was output at the end, using the PARSEVAL metric [Manning and Schütze (1999)]. Figure 5.3 shows how the F-score changes with the EM iterations when learning with each of four different curricula as well as in the no-curriculum baseline. It can be seen that learning with a curriculum consistently converges to a grammar with a better F-score than the no-curriculum baseline. Also, during the early stages of learning, the use of curricula results in faster improvements in F-score as compared to the no-curriculum baseline. The four curricula behave similarly, with the length-based curriculum using zero/one weights performing slightly better than the others.

We also plotted the change of rule probabilities during learning with a curriculum. Figure 5.4 shows the plot for VBD-headed grammar rules in learning with the length-based curriculum. The overall trends are very similar to those seen in Figure 5.2(c): the probability of each rule first rises and then drops, and rules are learned in a specific order. However, we can also see that some rules behave differently than specified by the curriculum, which is due to the errors or alternative parses made by the unsupervised learner. For example, the unsupervised learner learns to assign DT (determiner) as the head of a noun phrase, so in Figure 5.4 we see a curve for the rule VBD→DT, which is not present in Figure 5.2(c).

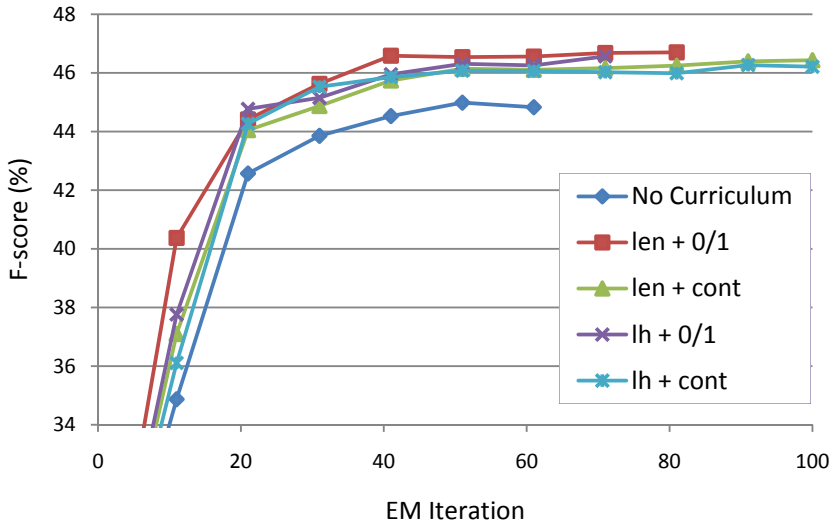


Figure 5.3 The change of F-scores with the EM iterations. "len" denotes length-based curriculum; "lh" denotes likelihood-based curriculum; "0/1" denotes that weights are set to be either zero or one; "cont" denotes that a continuous-valued weighting function is used in the weighting schemes.

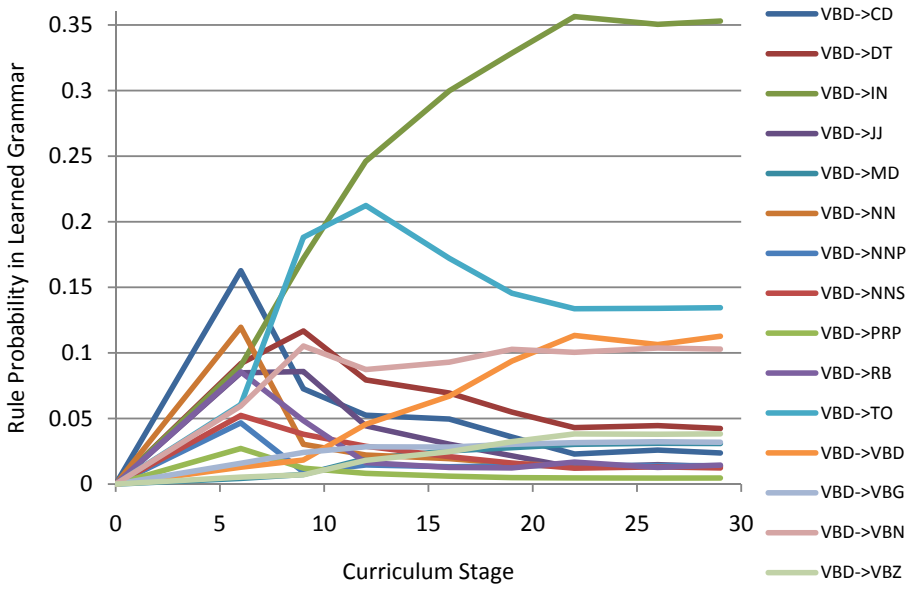


Figure 5.4 The change of probabilities of VBD-headed rules with the stages of the length--based curriculum during learning (best viewed in color). Rules with probabilities always below 0.025 are omitted.

5.6 Conclusion and Discussion

5.6.1 Related Work

Our work is motivated by the general curriculum learning framework proposed by Bengio et al. (2009) and the empirical work of applying curriculum learning to unsupervised grammar learning by Spitkovsky et al. (2010a). Kumar et al. (2010) proposed a special type of curriculum learning named self-paced learning, in which whether a training sample is included for training depends on the likelihood or risk of the sample given the learned parameters. Our likelihood-based curriculum with zero-or-one weights proposed in Section 5.5.2 can be seen as a special case of self-paced learning.

Curriculum learning is related to boosting algorithms in that both approaches learn from a weighted training set in a iterative fashion, with the weights being evolved from one iteration to the next. However, there are a few important differences between the two. First, boosting starts with a uniform weighting scheme and modifies the weights based on the learner's performance on the training data, whereas curriculum learning starts with a weighting scheme that favors easy samples and ends with a uniform weighting scheme. The easiness measure of training samples in curriculum learning is usually based on some external knowledge (e.g., the prior knowledge that shorter sentences are easier), which therefore introduces additional information into learning. In addition, in boosting we learn a set of base learners and then combine them by weighted voting, while in curriculum learning we continuously update a single learner.

Our likelihood-based curriculum learning is related to active learning, in that the choice of new training samples is based on the grammar that has been learned. The likelihood-based curriculum learning also resembles some self-training approaches, in that it re-weights training samples based on the probabilities of the samples given the learned grammar and such probabilities reflect the confidence of the learner in parsing the training samples.

5.6.2 Conclusion

We have provided an explanation of the benefits of curricula in the context of unsupervised learning of probabilistic grammars. Our explanation is based on the *incremental construction*

hypothesis which asserts that an ideal curriculum gradually emphasizes data samples that help the learner to successively discover new sub-structures of the target grammar. The hypothesis offers some guidance on the design of curricula as well as learning algorithms. We have presented results of experiments on synthetic data that provide support for the incremental construction hypothesis; we have further demonstrated the utility of curricula in unsupervised learning of grammars from a real-world natural language corpus.

CHAPTER 6. Conclusions

6.1 Summary

A grammar consists of a set of rules that specifies valid sentences of a language as well as the grammatical structures of such sentences. A probabilistic grammar augments the grammar rules with conditional probabilities and therefore defines a joint probability of a valid sentence and its grammatical structure. Probabilistic grammars have been used in many areas like natural language processing, bioinformatics, and pattern recognition, mainly for the purpose of deriving hidden grammatical structures from data (sentences).

Manually constructing a probabilistic grammar for a real-world application usually requires substantial human effort. Machine learning offers a potentially powerful approach to automatically inducing unknown grammars from data (a training corpus). Supervised grammar learning requires manual annotation of the grammatical structures of all the training sentences, which can be laborious and error-prone. Therefore, there is substantial interest in unsupervised grammar learning, which induces a grammar from unannotated sentences.

The existing approaches to unsupervised learning of probabilistic grammars can be divided into three categories. The first is structure search approaches, which try to find the optimal set of grammar rules along with the rule probabilities. The second is parameter learning approaches, which assume a fixed set of grammar rules and try to learn their probabilities. The third is incremental learning approaches, which specify a series of intermediate learning targets that culminate in the actual target grammar.

In this thesis we have focused on unsupervised learning of probabilistic context-free grammars and probabilistic dependency grammars. These two types of grammars are expressive enough to model many complicated real-world languages but remain tractable in inference. We

have presented three learning approaches, one in each of the three categories.

- The first approach is a structure search approach for learning probabilistic context-free grammars. It acquires rules of an unknown probabilistic context-free grammar through iterative coherent biclustering of the bigrams in the training corpus. A greedy procedure is used in our approach to add rules from biclusters such that each set of rules being added into the grammar results in the largest increase in the posterior of the grammar given the training corpus. Our experiments on several benchmark datasets have shown that this approach is competitive with existing methods for unsupervised learning of context-free grammars.

- The second approach is a parameter learning approach for learning natural language grammars based on the idea of *unambiguity regularization*. We made the observation that natural language is remarkably unambiguous in the sense that each natural language sentence has a large number of possible parses but only a few of the parses are syntactically valid. We incorporated this prior information into parameter learning by means of posterior regularization. The resulting algorithm is an extension of the expectation-maximization (EM) algorithm that is efficient and easy to implement. Both classic EM and Viterbi EM can be seen as a special case of our algorithm. Our experiments on real-world natural language data have shown that our algorithm has better performance than both classic EM and Viterbi EM.

- The third approach is learning with a curriculum, which is an incremental learning approach. A curriculum is a means of presenting training samples in a meaningful order. We introduced the *incremental construction hypothesis* that explains the benefits of a curriculum in learning grammars and offers some useful insights into the design of curricula as well as learning algorithms. We have presented results of experiments with (a) carefully crafted synthetic data that provide support for our hypothesis and (b) natural language corpus that demonstrate the utility of curricula in unsupervised learning of real-world probabilistic grammars.

6.2 Contributions

The main contributions of this thesis include:

- We provided a categorization of existing approaches to unsupervised learning of probabilistic grammars, and discussed the relations between these categories. In particular, we pointed out that parameter learning approaches are in general more scalable than structure search approaches, and that incremental approaches can potentially produce better results than the other two types of approaches in learning very complicated grammars.

- We proposed a novel structure search approach based on iterative biclustering. Our approach combines the advantages of heuristic structure search approaches and probabilistic structure search approaches. It is more rigorous and robust than previous heuristic approaches, and it is more efficient than previous probabilistic approaches.

- We introduced a novel regularization approach, the unambiguity regularization, for parameter learning of probabilistic natural language grammars. The resulting algorithm is efficient and easy to implement, and has good learning performance. The algorithm has both classic EM and Viterbi EM as its special cases, which also gives an explanation of the previously observed advantage of Viterbi EM in grammar learning. In addition to natural language grammar learning, the proposed unambiguity regularization may be useful in many other learning problems.

- We proposed the incremental construction hypothesis to explain the benefits of grammar learning with a curriculum. We gave both theoretical analysis and empirical evidence (on synthetic and real data) to support the hypothesis. Based on the hypothesis we offers some useful guidance on the design of curricula as well as learning algorithms. We also proposed a few novel curricula for learning probabilistic grammars.

6.3 Future Work

Some interesting directions for future work include:

- Although in Chapter 3 we have shown that our structure search approach based on iterative biclustering cannot scale up to real-world natural language data, there are a few directions to improve the approach. For example, since biclustering becomes unreliable on sparse and noisy data, we can learn grammar rules from multiple candidate biclusters instead of the best bicluster; and instead of greedily reducing training sentences with the new rules derived from the best bicluster, we can keep track of multiple candidate parses of each training sentence as a result of having multiple sets of candidate rules.

- As discussed in Chapter 4, one future research direction for our unambiguity regularization approach is to try other formulations of the ambiguity measurement and the regularization term. Another direction is to study how unambiguity regularization can be combined with other priors and regularizations for grammar learning. It would also be interesting to try unambiguity regularization on problems other than unsupervised natural language grammar learning.

- Structure search approaches are less scalable than parameter learning approaches, while parameter learning approaches have to assume a fixed grammar structure and can only remove grammar rules in learning (if parameter sparsity is encouraged). So a future direction is to combine these two types of approaches. Many existing structure search approaches do include a parameter learning component, but the purpose is merely to assign probabilities to grammar rules. One may employ the parameter learning component in structure search approaches to also refine the grammar structure, and only use the structure-changing operations for adding grammar rules or making large structure changes.

- We have experimented with a few different curricula in Chapter 5. There are some other curricula that are interesting to try. In particular, we may design a curriculum based on the real-world curriculum used by human parents and educators to teach languages to babies and children. For example, we can construct a curriculum that progressively includes sentences from children's readings of increasing age groups.

- In Chapter 5 we used the EM algorithm as the base learner in learning with a curriculum. We may employ other approaches, including those introduced in Chapter 3 and 4, as the base learner.

- The incremental learning approaches are relatively new and it is interesting to investigate approaches other than learning with a curriculum. For example, we can anneal the hyperparameters (of priors or regularization terms) in the objective function of parameter learning approaches to mimic the psychological development of children during language learning.

- In this thesis we have only tested our grammar learning approaches on language data. Considering the wide applications of probabilistic grammars, it is reasonable to test our approaches on other types of data, e.g., DNA/RNA sequence data and image data. Such data have different characteristics than the language data and therefore may demand additional changes to our approaches.

APPENDIX A. Derivations for the Structure Search Approach Based on Iterative Biclustering

In chapter 3 we have introduced a structure search approach based on iterative biclustering. In this appendix, we formulate how the learning process in the approach changes the posterior probability of the learned grammar given the training corpus.

The prior is defined as follows.

$$P(G) = 2^{-DL(G)}$$

where $DL(G)$ is the description length of the grammar G. In our algorithm we simply assume the same bit length for any symbol and use the length of the direct representation of the grammar as the description length, but other coding methods can also be used. This prior assigns higher probabilities to smaller grammars (the Occam's Razor principle). Since large, complex grammars are more likely to overfit the training corpus, we use this prior to prevent overfitting. This prior was also used in some previous Bayesian grammar learning algorithms Chen (1995).

To start with, we define a trivial initial grammar where the start symbol directly generates all the sentences in the training corpus. For each sentence $s_i =< w_1, w_2, \ldots, w_n >$ in the training corpus, where each w_j $(1 \leq j \leq n)$ is a terminal, the initial grammar contains the

following set of grammar rules.

$$
\begin{aligned}
&S \to w_1 S_{i1} \\
&S_{i1} \to w_2 S_{i2} \\
&S_{i2} \to w_3 S_{i3} \\
&\dots \\
&S_{i(n-2)} \to w_{n-1} S_{i(n-1)} \\
&S_{i(n-1)} \to w_n
\end{aligned}
$$

where S is the start symbol and each S_{ij} $(1 \le j \le n)$ is a nonterminal.

Starting from this initial grammar, our algorithm can be seen as gradually modifying it with the two steps described in the main text (the learning by biclustering step in Section 3.3.1 and the attaching step in Section 3.3.2), and we can formulate how such modifications change the posterior.

Notice that the formulation may be different if we use a different initial grammar (e.g., a CNF one), but as far as the initial grammar generates exactly the set of sentences in the training corpus, the difference should be limited to some constants in the formula and the conclusions should remain the same.

A.1 Learning a New AND-OR Group by Biclustering

In this section we formalize how the learning by biclustering step (Section 3.3.1 in the main text) changes the posterior. Suppose we extract a bicluster BC, create an AND symbol N and two OR symbols A, B, and add a set of grammar rules to the grammar:

$$
\begin{aligned}
&N \to AB \\
&A \to x \quad \text{for each row } x\text{, with the rule probability assigned} \\
&B \to y \quad \text{for each column } y\text{, with the rule probability assigned}
\end{aligned}
$$

We also reduce all the appearances of all the symbol pairs in BC to N in the corpus, and accordingly, we modify the grammar so that it generates these new "sentences" instead of the old ones. Specifically, for each appearance of each symbol pair xy in BC, in the original

grammar there are two rules

$$S_{ij} \to xS_{i(j+1)}$$
$$S_{i(j+1)} \to yS_{i(j+2)}$$

which are now combined into

$$S_{ij} \to NS_{i(j+2)}$$

First, let's look at how the likelihood is changed. For each sentence that's involved in the reduction, its likelihood is changed by two factors. First, the original derivation that generates xy now generates N instead, and then N generates xy with the probability $P(A \to x|A) \times P(B \to y|B)$; so the likelihood of this sentence is reduced by a factor equal to this probability. Second, the reduction may make some other sentences in the corpus become the same as this sentence, so the likelihood is increased by a factor equal to how many times the number of such equivalent sentences increases. Suppose this sentence is represented by row p and column q in the expression-context matrix, then this second factor is exactly the ratio of the sum of column q to the value of cell (p, q), because before the reduction only those sentences represented by that cell are equivalent, and after the reduction the sentences in the whole column become equivalent (the same context plus the same expression N). To sum up, we can formalize the likelihood gain resulted from the grammar modification as follows.

Denote the likelihood gain of extracting BC by $LG(BC)$. Let D be the set of sentences in the training corpus, and let G_k and G_{k+1} be the grammar before and after extracting the bicluster. By abuse of notation we denote the set of rows of BC by A, and the set of columns by B, and denote the context of a symbol pair xy in a sentence d by $d-$"xy". For the bicluster BC, denote the sum of row x by r_x, the sum of column y by c_y. For the expression-context matrix, denote its value at row i and column j by $EC(i, j)$, its set of rows by EC-row, its set of columns by EC-col, and the sum of column q by c'_q.

$$\begin{aligned} LG(BC) &= \frac{P(D|G_{k+1})}{P(D|G_k)} = \prod_{d \in D} \frac{P(d|G_{k+1})}{P(d|G_k)} \\ &= \prod_{x \in A,\ y \in B,\ xy \text{ appears in } d \in D} P(x|A)P(y|B)\frac{\sum_{p \in \text{EC-row}} EC(p, d - \text{“xy”})}{EC(\text{“xy”}, d - \text{“xy”})} \\ &= \prod_{x \in A} P(x|A)^{r_x} \prod_{y \in B} P(y|B)^{c_y} \frac{\prod_{q \in \text{EC-col}} {c'_q}^{c'_q}}{\prod_{\substack{p \in \text{EC-row} \\ q \in \text{EC-col}}} EC(p, q)^{EC(p,q)}} \end{aligned} \tag{A.1}$$

To maximize the likelihood gain, $P(x|A)$ and $P(y|B)$ must take the following form, which can be obtained by applying the Lagrange multiplier method with these two sets of probabilities as the variables.

$$P(x|A) = \frac{r_x}{s} \tag{A.2}$$

$$P(y|B) = \frac{c_y}{s} \tag{A.3}$$

where s is the sum of all the cells in BC. This form is also what one would intuitively expect.

Putting it into the likelihood gain formula, we get

$$\begin{aligned} \max_{P_r} LG(BC) &= \prod_{x\in A}\left(\frac{r_x}{s}\right)^{r_x}\prod_{y\in B}\left(\frac{c_y}{s}\right)^{c_y}\frac{\prod_{q\in\text{EC-col}} {c'_q}^{c'_q}}{\prod_{\substack{p\in\text{EC-row}\\ q\in\text{EC-col}}} EC(p,q)^{EC(p,q)}} \\ &= \frac{\prod_{x\in A} {r_x}^{r_x}\prod_{y\in B} {c_y}^{c_y}}{s^{2s}} \times \frac{\prod_{q\in\text{EC-col}} {c'_q}^{c'_q}}{\prod_{\substack{p\in\text{EC-row}\\ q\in\text{EC-col}}} EC(p,q)^{EC(p,q)}} \end{aligned} \tag{A.4}$$

where P_r represents the set of grammar rule probabilities.

Let a_{xy} be the cell value at row x and column y of the bicluster BC; for the expression-context matrix, let s' be the sum of all values in the matrix, and let r'_p be the sum of row p. Notice that $s = s'$ and $a_{xy} = r'_p$ (where row p of the expression-context matrix represents the symbol pair xy). So we can get

$$\begin{aligned} &\max_{P_r} LG(BC) \\ &= \frac{\prod_{x\in A} {r_x}^{r_x}\prod_{y\in B} {c_y}^{c_y}}{s^s} \times \left(\frac{\prod_{p\in\text{EC-row}} {r'_p}^{r'_p}}{\prod_{\substack{x\in A\\ y\in B}} {a_{xy}}^{a_{xy}}}\right) \times \frac{\prod_{q\in\text{EC-col}} {c'_q}^{c'_q}}{s'^{s'}\prod_{\substack{p\in\text{EC-row}\\ q\in\text{EC-col}}} EC(p,q)^{EC(p,q)}} \\ &= \frac{\prod_{x\in A} {r_x}^{r_x}\prod_{y\in B} {c_y}^{c_y}}{s^s\prod_{\substack{x\in A\\ y\in B}} {a_{xy}}^{a_{xy}}} \times \frac{\prod_{p\in\text{EC-row}} {r'_p}^{r'_p}\prod_{q\in\text{EC-col}} {c'_q}^{c'_q}}{s'^{s'}\prod_{\substack{p\in\text{EC-row}\\ q\in\text{EC-col}}} EC(p,q)^{EC(p,q)}} \end{aligned} \tag{A.5}$$

It can be seen that the two factors in Eq.A.5 are of the same form, one for the bicluster and one for the expression-context matrix. Indeed, this form of formula measures the multiplicative coherence of the underlying matrix, in a similar way as in Madeira and Oliveira (2004). It reaches the maximum of 1 iff. the underlying matrix has perfect multiplicative coherence (easy to prove by using the Lagrange multiplier method). Therefore we get the conclusion that, by extracting a bicluster, the maximal likelihood gain is the product of the multiplicative coherence of the bicluster and the multiplicative coherence of its expression-context matrix.

Notice that in the above formalization, we don't consider the possibility that some sentences containing xy ($x \in A$, $y \in B$) may have been reduced before we learn this AND-OR group. In that case, when this new AND-OR group is learned, we may get new ways of parsing such sentences, thus increasing their likelihood. So when there's significant ambiguity in the target grammar, the above formulas may not give an accurate estimation of the real likelihood gain.

Now let's turn to the prior, which is solely determined by the grammar size. By extracting a bicluster, a set of new rules are added into the grammar, which has $4+(2+2|A|)+(2+2|B|)$ symbols. On the other hand, each reduction (of xy to N) decreases the grammar size by 4 symbols, and there are $s = \sum_{x\in A, y\in B} a_{xy}$ number of such reductions. So overall,

$$\frac{P(G_{k+1})}{P(G_k)} = \frac{2^{-(DL(G_K)+(4+(2+2|A|)+(2+2|B|))\alpha-4s\alpha)}}{2^{-DL(G_k)}} = 2^{\alpha(4s-2|A|-2|B|-8)} \tag{A.6}$$

where α is the number of bits needed to represent a symbol.

Combining Eq.A.5 and Eq.A.6, we can get the posterior gain formula when extracting a bicluster (with the optimal grammar rule probabilities assigned), as shown in Eq.3.2 in the main text. Notice that Eq.A.2 and A.3 still hold for maximizing the posterior gain, because the values of $P(x|A)$ and $P(y|B)$ don't have any effect on the prior gain.

A.2 Attaching the New AND Symbol under Existing OR Symbols

In this section we try to formalize how the attaching step (Section 3.3.2 in the main text) changes the posterior. Suppose we add a new rule $O \to N$ into the grammar G_k, and do a maximal reduction on the involved sentences, resulting in a new grammar G_{k+1}. Suppose O was learned by extracting the bicluster BC, together with M and P s.t. $M \to OP$ (the following derivation can also be applied to $M \to PO$). So O corresponds to the set of rows of BC and P corresponds to the set of columns. By adding the rule $O \to N$, we expand BC by adding a new row, which records the appearance number of Ny in the corpus for each $y \in P$. Let EC be the expression-context matrix of BC, and EC-row and EC-col be the set of rows and columns of EC. With the new rule $O \to N$, EC is also expanded with a set of new rows for the new expressions containing N, and we use $EC(\text{“Ny”}, q)$ to represent the value at the new row Ny and column q in the expanded expression-context matrix. Because we may change

the rule probabilities after adding the new rule, denote the original rule probabilities by $P()$ and the new rule probabilities by $P'()$.

The likelihood is changed in the following way. First, for the sentences involved in the new row for N, each appearance of $Ny\,(y \in P)$ is reduced to M, leading to a likelihood change just as discussed in the previous section. Second, for the sentences involved in BC, since we change the probabilities of rules related to BC, and the reduction of Ny to M results in more equivalent sentences, their likelihood is changed accordingly.

$$\begin{aligned}
&\frac{P(D|G_{k+1})}{P(D|G_k)} \\
&= \prod_{y\in P,\ Ny \text{ appears in } d\in D} \left(P'(N|O)P'(y|P)\frac{\widetilde{col}(d - \text{“Ny”})}{EC(\text{“Ny”}, d - \text{“Ny”})} \right) \\
&\quad \times \prod_{x\in O,\ y\in P,\ xy \text{ appears in } d\in D} \left(\frac{P'(x|O)P'(y|P)}{P(x|O)P(y|P)} \times \frac{\widetilde{col}(d - \text{“xy”})}{\sum_{p\in \text{EC-row}} \widetilde{EC}(p, d - \text{“xy”})} \right)
\end{aligned} \tag{A.7}$$

where $\widetilde{col}()$ is defined as

$$\widetilde{col}(cont) = \sum_{p\in \text{EC-row}} \widetilde{EC}(p, cont) + \sum_{z\in P} EC(\text{“Nz”}, cont) \tag{A.8}$$

$\widetilde{EC}(p, q)$ represents the value of cell pq in the *derived* expression-context matrix, which is the expected appearance number of the combination of expression p and context q when the current learned grammar G_k is applied to expand the current partially reduced corpus. To construct $\widetilde{EC}$, we have to enumerate all the AND symbols that M may be directly or indirectly reduced to, and traverse their appearances in the partially reduced corpus. Based on the definition of $\widetilde{EC}$, it's obvious that it is perfectly multiplicatively coherent.

Let $\widetilde{EC}'$ be the expanded derived expression-context matrix containing both $\widetilde{EC}$ and the new rows for Ny $(y \in P)$. So $\widetilde{col}(q)$ is the sum of column q in $\widetilde{EC}'$. Let EC-row$'$ and EC-col$'$ be the set of rows and columns of $\widetilde{EC}'$. Let EC' be the actual expanded expression-context matrix containing both EC and the new rows. Let $col(q)$ be the sum of column q in EC'.

Let $\widetilde{BC}$ be the derived bicluster that records the expected appearance number of each symbol pair xy $(x \in O, y \in P)$ when applying the current learned grammar G_k to expand the

current partially reduced corpus. So $\widetilde{EC}$ is its expression-context matrix. It can be proved that, when recursive rules are not involved in generating symbol pairs in $\widetilde{BC}$, it has the same row sums, column sums and total sum as BC, but its cell values may be different, which makes it perfectly multiplicatively coherent. When recursive rules are involved, however, $\widetilde{BC}$ and BC might be quite different. Let r_x be the sum of row x and c_y be the sum of column y in $\widetilde{BC}$. Let $\widetilde{BC}'$ be the expanded derived bicluster that contains both $\widetilde{BC}$ and the new row for N. Let r_N be the sum of the new row for N, and a_{Ny} be the cell value at column y in the new row.

Since $P(x|O)P(y|P)\sum_{p\in\text{EC-row}}\widetilde{EC}(p, d-\text{“xy”}) = \widetilde{EC}(\text{“xy”}, d-\text{“xy”})$, we may further reduce Eq.A.7 as follows.

$$\frac{P(D|G_{k+1})}{P(D|G_k)} = \prod_{x\in O\cup\{N\}} P'(x|O)^{r_x} \prod_{y\in P} P'(y|P)^{c_y+a_{Ny}} \frac{\prod_{q\in\text{EC-col}'} \widetilde{col}(q)^{col(q)}}{\prod_{\substack{p\in\text{EC-row}'\\ q\in\text{EC-col}'}} \widetilde{EC}'(p,q)^{EC'(p,q)}} \tag{A.9}$$

It can be proved that, if for every AND-OR group involved in calculating $\widetilde{EC}$, the bicluster and its expression-context matrix are both perfectly multiplicatively coherent, and if no recursive rules are involved, then $EC = \widetilde{EC}$. Since we learn new rules only when Eq.3.2 or Eq.3.3 is large enough, we expect that the likelihood gain of each step in the algorithm is close to the maximum of 1 and thus the biclusters and their expression-context matrixes are approximately multiplicatively coherent. So we use $\widetilde{EC}$ to approximate EC and use $\widetilde{col}(q)$ to approximate $col(q)$, and therefore according to Eq.A.1 we get

$$\frac{P(D|G_{k+1})}{P(D|G_k)} \approx LG(\widetilde{BC}') \tag{A.10}$$

Again it can be shown that for the new set of rule probabilities P_r, Eq.A.2 and A.3 hold when the likelihood gain is maximized. In addition we know both $\widetilde{BC}$ and $\widetilde{EC}$ are perfectly multiplicatively coherent. So we get

$$\begin{aligned}\max_{P_r} \frac{P(D|G_{k+1})}{P(D|G_k)} &\approx \max_{P_r} LG(\widetilde{BC}')\\ &= \frac{f(r_N)\times\prod_{y\in P} f(c_y+a_{Ny})\times f(s)^2\times\prod_{q\in\text{EC-col}} f\left(c'_q+\sum_{y\in P} EC(\text{“Ny”},q)\right)}{\prod_{y\in P} f(c_y)\times f(s+r_N)^2\times\prod_{\substack{y\in P\\ q\in\text{EC-col}}} f(EC(\text{“Ny”},q))\times\prod_{q\in\text{EC-col}} f(c'_q)}\end{aligned} \tag{A.11}$$

where $f(x) = x^x$; s is the total sum of $\widetilde{BC}$; c'_q is the sum of column q of $\widetilde{EC}$.

Notice that there might be a third part in the likelihood change in addition to the two discussed above: after Ny is reduced to M, it may be further reduced, leading to a series of likelihood changes. However, it can be proved that if 1) Eq.A.11 reaches its maximum of 1, i.e., $\widetilde{BC}'$ and its expression-context matrix are perfectly multiplicatively coherent, and 2) for every AND-OR group involved in calculating $\widetilde{EC}$, the bicluster and its expression-context matrix are both perfectly multiplicatively coherent, then the likelihood gain caused by this part is 1, i.e., the likelihood is not changed. Since we learn new rules only when Eq.3.2 or Eq.3.3 is large enough, we expect that both conditions are approximately satisfied and the likelihood change caused by this part is small. Besides, we have to do maximal reduction to calculate the effect of this part, which would be too time-consuming if we do it for every candidate new rule. So we choose to omit this part.

Now let's turn to the prior. There are two changes of the grammar length. First, a new rule $O \to N$ is added into the grammar. Second, we reduce $Ny\,(y \in P)$ to M, so in the grammar, for each appearance of Ny, the two rules that generate N and y are now combined to one that generates M. Therefore the prior gain is

$$\frac{P(G_{k+1})}{P(G_k)} = \frac{2^{-(DL(G_K)+2\alpha-4r_N\alpha)}}{2^{-DL(G_k)}} = 2^{\alpha(4r_N-2)} \tag{A.12}$$

where r_N is the sum of the new row for N in $\widetilde{BC}'$. Again, here we omit the changes caused by possible further reductions after Ny is reduced to M.

Putting Eq.A.11 and A.12 together, we get the approximate posterior gain when learning a new rule in the attaching step (with the optimal grammar rule probabilities assigned). It's easy to see that the result is equal to the ratio of the maximal posterior gain by extracting $\widetilde{BC}'$ to the maximal posterior gain by extracting $\widetilde{BC}$, as shown in Eq.3.3 of the main text.

APPENDIX B. Proofs for the Parameter Learning Approach with Unambiguity Regularization

B.1 Theorem Proofs in Case 2: $0 < \sigma < 1$.

To prove Theorem 4.1, we first prove the following lemma.

Lemma B.1. *$x \log x$ is strictly convex on the interval [0,1], where $0 \log 0$ is defined to be 0.*

Proof. On the interval (0,1], $x \log x$ is twice differentiable and we have

$$\frac{\partial^2 x \log x}{\partial x^2} = \frac{1}{x} > 0$$

So $x \log x$ is strictly convex on the interval (0,1]. Now note that

$$\begin{aligned} \forall x \in (0,1], \forall t \in (0,1), \quad (0t + (1-t)x)\log(0t + (1-t)x) &= (1-t)x \log(1-t)x \\ &< (1-t)x \log x = t \times 0 \log 0 + (1-t)x \log x \end{aligned}$$

Therefore $x \log x$ is also strictly convex with 0 included in the interval. □

Now we prove Theorem 4.1.

Theorem 4.1. *$f_i(q)$ is strictly convex on the unit simplex Δ.*

Proof. For any $t \in (0,1)$, for any two points q_1 and q_2 in the unit simplex Δ, we need to prove that $f_i(tq_1 + (1-t)q_2) < tf_i(q_1) + (1-t)f_i(q_2)$. The left-hand side is

$$\begin{aligned} f_i(tq_1 + (1-t)q_2) &= \sum_{z_i} \left[(tq_1(z_i) + (1-t)q_2(z_i)) \log \frac{(tq_1(z_i) + (1-t)q_2(z_i))^{1-\sigma}}{p_\theta(z_i|x_i)} \right] \\ &= \sum_{z_i} [(1-\sigma)(tq_1(z_i) + (1-t)q_2(z_i)) \log (tq_1(z_i) + (1-t)q_2(z_i)) \\ &\quad - (tq_1(z_i) + (1-t)q_2(z_i)) \log p_\theta(z_i|x_i)] \end{aligned}$$

The right-hand side is

$$\begin{aligned} tf_i(q_1) + (1-t)f_i(q_2) &= \sum_{z_i}\left[tq_1(z_i)\log\frac{q_1(z_i)^{1-\sigma}}{p_\theta(z_i|x_i)} + (1-t)q_2(z_i)\log\frac{q_2(z_i)^{1-\sigma}}{p_\theta(z_i|x_i)}\right] \\ &= \sum_{z_i}[(1-\sigma)tq_1(z_i)\log q_1(z_i) + (1-\sigma)(1-t)q_2(z_i)\log q_2(z_i) \\ &\quad - (tq_1(z_i) + (1-t)q_2(z_i))\log p_\theta(z_i|x_i)] \end{aligned}$$

So the right-hand side minus the left-hand side is

$$\begin{aligned} & tf_i(q_1) + (1-t)f_i(q_2) - f_i(tq_1 + (1-t)q_2) \\ = \;& (1-\sigma)\sum_{z_i}[tq_1(z_i)\log q_1(z_i) + (1-t)q_2(z_i)\log q_2(z_i) \\ & - (tq_1(z_i) + (1-t)q_2(z_i))\log(tq_1(z_i) + (1-t)q_2(z_i))] \end{aligned}$$

Because $\forall z_i, 0 \le q_1(z_i), q_2(z_i) \le 1$, Lemma B.1 implies that

$$tq_1(z_i)\log q_1(z_i) + (1-t)q_2(z_i)\log q_2(z_i) > (tq_1(z_i) + (1-t)q_2(z_i))\log(tq_1(z_i) + (1-t)q_2(z_i))$$

So we have

$$tf_i(q_1) + (1-t)f_i(q_2) - f_i(tq_1 + (1-t)q_2) > 0$$

□

B.2 Theorem Proofs in Case 4: $\sigma > 1$.

We first introduce the following theorem.

Theorem B.1. *$f_i(q)$ is strictly concave on the unit simplex Δ.*

The proof is the same as that of theorem 4.1, except that $1-\sigma$ is now negative which reverses the direction of the last inequality in the proof.

Now we can prove Theorem 4.2.

Theorem 4.2. *The minimum of $f_i(q)$ is attained at a vertex of the unit simplex Δ.*

Proof. Assume the minimum of $f_i(q)$ is attained at q^* that is not a vertex of the unit simplex Δ, so there are at least two assignments of z_i, say z^1 and z^2, such that $q^*(z^1)$ and $q^*(z^2)$ are nonzero. Let q' be the same distribution as q^* except that $q'(z^1) = 0$ and $q'(z^2) = q^*(z^1) + q^*(z^2)$. Let q''

be the same distribution as q^* except that $q''(z^1) = q^*(z^1) + q^*(z^2)$ and $q''(z^2) = 0$. Obviously, both q' and q'' are in the unit simplex Δ and $q' \neq q''$. Let $t = \frac{q^*(z^2)}{q^*(z^1)+q^*(z^2)}$, and obviously we have $0 < t < 1$. So we get $q^* = tq' + (1-t)q''$. According to Theorem B.1, $f_i(q)$ is strictly concave on the unit simplex Δ, so we have $f_i(q^*) > tf_i(q') + (1-t)f_i(q'')$. Without loss of generality, suppose $f_i(q') \geq f_i(q'')$. So we have $tf_i(q') + (1-t)f_i(q'') \geq f_i(q'')$ and therefore $f_i(q^*) > f_i(q'')$, which means $f_i(q)$ does not attain the minimum at q^*. This contradicts the assumption. □

APPENDIX C. Supplementary Material for the Incremental Learning Approach by Using Curricula

Section C.1 provides the proofs of the theorems in section 5.3. Section C.2 gives more details of the experimental settings in section 5.4 and 5.5.

C.1 Proofs of Theorems

We first prove Theorem 5.1.

Theorem 5.1. *If a curriculum $\langle W_1, W_2, \ldots, W_n \rangle$ satisfies incremental construction (with either condition 3 or 3b), then for any i, j, k s.t. $1 \leq i < j < k \leq n$, we have*

$$\begin{aligned} d_1(\theta_i, \theta_k) &\geq d_1(\theta_j, \theta_k) \\ d_{TV}(G_i, G_k) &\geq d_{TV}(G_j, G_k) \end{aligned}$$

where $d_1(\cdot, \cdot)$ denotes the L_1 distance; $d_{TV}(G_i, G_j)$ represents the total variation distance between the two distributions of grammatical structures defined by G_i and G_j.

Proof. Here we assume condition 3b because it is more general than condition 3. There are two inequalities in the conclusion of the theorem. We first give the proof of the inequality with the L_1 distance of parameter vectors. As defined in Section 5.3, a parameter vector is the concatenation of a set of multinomial vectors, each of which is the vector of probabilities of grammar rules with a specific rule condition (left-hand side) of the target grammar. Denote $\theta_{i,p}$ as the multinomial vector of rule condition p in grammar G_i, and denote $\theta_{i,p \to q}$ as the probability of rule $p \to q$ in grammar G_i. Note that

$$d_1(\theta_i, \theta_j) = \sum_p d_1(\theta_{i,p}, \theta_{j,p})$$

So to prove the first inequality, it is sufficient to prove that

$$\forall p,\ d_1(\theta_{i,p}, \theta_{k,p}) \geq d_1(\theta_{j,p}, \theta_{k,p})$$

Because the L_1 norm of a multinomial vector is always 1, for any rule condition p we have

$$\begin{aligned}
d_1(\theta_{i,p}, \theta_{j,p}) &= \sum_{q:\theta_{i,p\to q} > \theta_{j,p\to q}} (\theta_{i,p\to q} - \theta_{j,p\to q}) + \sum_{q:\theta_{i,p\to q} \leq \theta_{j,p\to q}} (\theta_{j,p\to q} - \theta_{i,p\to q}) \\
&= \left(1 - \sum_{q:\theta_{i,p\to q} \leq \theta_{j,p\to q}} \theta_{i,p\to q}\right) - \left(1 - \sum_{q:\theta_{i,p\to q} \leq \theta_{j,p\to q}} \theta_{j,p\to q}\right) \\
&\quad + \sum_{q:\theta_{i,p\to q} \leq \theta_{j,p\to q}} (\theta_{j,p\to q} - \theta_{i,p\to q}) \\
&= 2 \times \sum_{q:\theta_{i,p\to q} \leq \theta_{j,p\to q}} (\theta_{j,p\to q} - \theta_{i,p\to q}) \\
&= 2 \times \sum_{q} (\theta_{j,p\to q} - \theta_{i,p\to q}) f_{i,j,p}(q) \qquad \text{(C.1)}
\end{aligned}$$

where $f_{i,j,p}(q)$ is defined as

$$f_{i,j,p}(q) = \begin{cases} 1 & \text{if } \theta_{i,p\to q} \leq \theta_{j,p\to q} \\ 0 & \text{if } \theta_{i,p\to q} > \theta_{j,p\to q} \end{cases}$$

According to Definition 5.1 (with condition 3b), for any grammar rule $p \to q$ in the target grammar, with the increase of i, its probability $\theta_{i,p\to q}$ first remains 0, then shifts to a non-zero value in a certain intermediate grammar, and after that decreases monotonically. So for any $i < j < k$, there are three possibilities, which we consider in turn.

1. If $\theta_{i,p\to q} = \theta_{j,p\to q} = 0$ and $\theta_{k,p\to q} \geq 0$, then we have

$$(\theta_{k,p\to q} - \theta_{i,p\to q}) f_{i,k,p}(q) = (\theta_{k,p\to q} - \theta_{j,p\to q}) f_{j,k,p}(q)$$

2. If $\theta_{i,p\to q} = 0$ and $\theta_{j,p\to q} \geq \theta_{k,p\to q} > 0$, then we have

$$(\theta_{k,p\to q} - \theta_{i,p\to q}) f_{i,k,p}(q) > 0 = (\theta_{k,p\to q} - \theta_{j,p\to q}) f_{j,k,p}(q)$$

3. If $\theta_{i,p\to q} \geq \theta_{j,p\to q} \geq \theta_{k,p\to q} > 0$, then we have

$$(\theta_{k,p\to q} - \theta_{i,p\to q}) f_{i,k,p}(q) = (\theta_{k,p\to q} - \theta_{j,p\to q}) f_{j,k,p}(q) = 0$$

Therefore, we get

$$\sum_q(\theta_{k,p\to q}-\theta_{i,p\to q})f_{i,k,p}(q)\geq\sum_q(\theta_{k,p\to q}-\theta_{j,p\to q})f_{j,k,p}(q)$$

where equality holds if there exists no assignment of q that satisfies the second possibility. According to Eq.C.1, we have

$$d_1(\theta_{i,p},\theta_{k,p})\geq d_1(\theta_{j,p},\theta_{k,p})$$

Therefore we have proved the first inequality.

Now we turn to the second inequality in the conclusion of the theorem and prove it in a similar fashion. Because the sum of probabilities over all grammatical structures is always 1, we have

$$\begin{aligned} d_{TV}(G_i,G_j) &= \frac{1}{2}\sum_s|P(s|G_i)-P(s|G_j)| \\ &= \sum_s(P(s|G_j)-P(s|G_i))f_{i,j}(s) \end{aligned} \tag{C.2}$$

where $f_{i,j}(s)$ is defined as

$$f_{i,j}(s)=\begin{cases} 1 & \text{if } P(s|G_i)\leq P(s|G_j) \\ 0 & \text{if } P(s|G_i)>P(s|G_j) \end{cases}$$

The first equality of Eq.C.2 is the definition of total variation, and the second equality can be derived in a similar way as in Eq.C.1. According to Definition 5.1 (with condition 3b), for any grammatical structure s that can be generated by the target grammar, with the increase of i, the probability $P(s|G_i)$ first remains 0 (when at least one grammar rule used in deriving s is absent from G_i), then shifts to a non-zero value (when all the grammar rules needed to derive s have non-zero probabilities), and after that decreases monotonically (because the probabilities of all the grammar rules used in deriving s are decreasing). Just as in the proof of the first inequality, for any $i<j<k$ there are three possibilities, and by analyzing the three possibilities in turn we can get

$$\sum_s(P(s|G_k)-P(s|G_i))f_{i,k}(s)\geq\sum_s(P(s|G_k)-P(s|G_j))f_{j,k}(s)$$

So according to Eq.C.2, we have

$$d_{TV}(G_i, G_k) \geq d_{TV}(G_j, G_k)$$

Therefore we have proved the second inequality. □

Now we give a proof sketch of Theorem 5.2.

Theorem 5.2. *If a curriculum $\langle W_1, W_2, \ldots, W_n \rangle$ satisfies the first two conditions in Definition 5.1 as well as a further relaxed version of the third condition:*

3c. for any grammar rules r, $P(r|G_i)$ first monotonically increases with i and then monotonically decreases with i.

then for any i, j, k s.t. $1 \leq i < j < k \leq n$, we have

$$d_1(\theta_i, \theta_k) \geq d_1(\theta_j, \theta_k)$$

Proof Sketch. The proof is the same as the proof of the first inequality of Theorem 5.1, except that the three possibilities are changed because of condition 3c. According to condition 3c, with the increase of i, the probability of a grammar rule $\theta_{i,p\to q}$ first increases monotonically and then decreases monotonically. So for any $i < j < k$, we have three new possibilities.

1. If $\theta_{i,p\to q} \leq \theta_{j,p\to q} \leq \theta_{k,p\to q}$, then we have

$$(\theta_{k,p\to q} - \theta_{i,p\to q}) f_{i,k,p}(q) \geq (\theta_{k,p\to q} - \theta_{j,p\to q}) f_{j,k,p}(q)$$

2. If $\theta_{i,p\to q} \leq \theta_{j,p\to q}$ and $\theta_{j,p\to q} \geq \theta_{k,p\to q}$, then we have

$$(\theta_{k,p\to q} - \theta_{i,p\to q}) f_{i,k,p}(q) \geq 0 = (\theta_{k,p\to q} - \theta_{j,p\to q}) f_{j,k,p}(q)$$

3. If $\theta_{i,p\to q} \geq \theta_{j,p\to q} \geq \theta_{k,p\to q}$, then we have

$$(\theta_{k,p\to q} - \theta_{i,p\to q}) f_{i,k,p}(q) = (\theta_{k,p\to q} - \theta_{j,p\to q}) f_{j,k,p}(q) = 0$$

So we can still get

$$\sum_q (\theta_{k,p\to q} - \theta_{i,p\to q}) f_{i,k,p}(q) \geq \sum_q (\theta_{k,p\to q} - \theta_{j,p\to q}) f_{j,k,p}(q)$$

and the rest of the proof is exactly the same as in the proof of the first inequality of Theorem 5.1. □

C.2 Experiments

In this section we provide more details of the experiments presented in section 5.4 and 5.5.

We adapted the DAGEEM software[1] to implement the expectation-maximization algorithm of the DMV grammar. We then implemented curriculum learning by using expectation-maximization as the base learner. In the experiments on synthetic data, expectation-maximization was initialized with a trivial grammar in which rules with the same left-hand side have equal probabilities; in the experiments on real data, we used an initial grammar provided in the DAGEEM software which is created according to the heuristic approach described in Klein and Manning (2004). As mentioned in section 5.4, we used a dynamic smoothing factor in the experiments on synthetic data to alleviate the overfitting problem discussed in Section 5.3.1. The dynamic smoothing factor is computed from the size of the partial corpus that is hidden from the learner during curriculum learning. More specifically, for each training sentence that is hidden, we assume it is sampled from a uniform distribution over all possible sentences that have the same length as the hidden sentence, and we also assume a uniform grammar in which rules with the same left-hand side have equal probabilities, so we can easily compute the expected counts of each grammar rule r being used in parsing this sentence; then the dynamic smoothing factor for grammar rule r is the sum of the expected counts over all the training sentences that are hidden. We found that dynamic smoothing often improves the learning result in the experiments on synthetic data; however, in the experiments on real data, dynamic smoothing usually hurts learning.

We used the WSJ30 corpus (the set of sentences no longer than 30 in the Wall Street Journal corpus of the Penn Treebank) in our experiments. Because we used the DMV grammar formalism in our experiments, which is a type of dependency grammar, we converted the phrase structure annotations in the Penn Treebank to the dependency annotations by running the "ptbconv" software[2]. When generating the synthetic data, we found the dependency treebank grammar of WSJ30 tends to generate sentences much longer than the actual sentences in WSJ30, so we decreased by 30% the probabilities of grammar rules that determine if a new

[1] http://www.ark.cs.cmu.edu/DAGEEM/

[2] Available at http://www.jaist.ac.jp/~h-yamada/

dependency should be generated under a certain condition.

We tested different values of the smoothing factor in the experiments on both synthetic data and real data. We found that although the value of the smoothing factor did affect the learning performance, the advantage of curriculum learning over the baseline was consistently observed.

BIBLIOGRAPHY

Adriaans, P., Trautwein, M., and Vervoort, M. (2000). Towards high speed grammar induction on large text corpora. In *SOFSEM 2000, LNCS 1963*.

Baker, J. K. (1979). Trainable grammars for speech recognition. In *Speech Communication Papers for the 97th Meeting of the Acoustical Society of America*.

Baum, L. E., Petrie, T., Soules, G., and Weiss, N. (1970). A maximization technique occurring in the statistical analysis of probabilistic functions of markov chains. *Ann. Math. Statist.*, 41(1):164C171.

Bengio, Y., Louradour, J., Collobert, R., and Weston, J. (2009). Curriculum learning. In *ICML*, page 6.

Bod, R. (2006). An all-subtrees approach to unsupervised parsing. In *Proceedings of ACL*.

Charniak, E. (1996). Tree-bank grammars. In *AAAI/IAAI, Vol. 2*, pages 1031–1036.

Charniak, E. (1997). Statistical parsing with a context-free grammar and word statistics. In *AAAI/IAAI*, pages 598–603.

Chen, S. F. (1995). Bayesian grammar induction for language modeling. In *Proceedings of the 33rd annual meeting on Association for Computational Linguistics*.

Clark, A. (2001). Unsupervised induction of stochastic context-free grammars using distributional clustering. In *Proceedings of CoNLL*.

Clark, A. (2007). Learning deterministic context free grammars: The omphalos competition. *Machine Learning*, 66.

Clark, A., Eyraud, R., and Habrard, A. (2008). A polynomial algorithm for the inference of context free languages. In *ICGI*, pages 29–42.

Cohen, S. B., Gimpel, K., and Smith, N. A. (2008). Logistic normal priors for unsupervised probabilistic grammar induction. In *NIPS*, pages 321–328.

Cohen, S. B. and Smith, N. A. (2009). Shared logistic normal distributions for soft parameter tying in unsupervised grammar induction. In *HLT-NAACL*, pages 74–82.

Cohn, T., Goldwater, S., and Blunsom, P. (2009). Inducing compact but accurate treesubstitution grammars. In *In Proc. NAACL*.

Collins, M. J. (1999). *Head-driven statistical models for natural language parsing*. PhD thesis, Philadelphia, PA, USA. Supervisor-Marcus, Mitchell P.

Cramer, B. (2007). Limitations of current grammar induction algorithms. In *Proceedings of the 45th Annual Meeting of the ACL: Student Research Workshop*, ACL '07, pages 43–48, Stroudsburg, PA, USA. Association for Computational Linguistics.

Daumé, III, H. (2009). Unsupervised search-based structured prediction. In *ICML*, page 27.

de la Higuera, C. (2005). A bibliographical study of grammatical inference. *Pattern Recogn.*, 38(9):1332–1348.

Durbin, R., Eddy, S. R., Krogh, A., and Mitchison, G. J. (1998). *Biological Sequence Analysis: Probabilistic Models of Proteins and Nucleic Acids*. Cambridge University Press.

Elman, J. L. (1993). Learning and development in neural networks: The importance of starting small. *Cognition*, 48:71–99.

Finkel, J. R., Grenager, T., and Manning, C. D. (2007). The infinite tree. In *Proceedings of the 45th Annual Meeting of the Association of Computational Linguistics*, pages 272–279. Association for Computational Linguistics.

Ganchev, K., Graça, J., Gillenwater, J., and Taskar, B. (2010). Posterior regularization for structured latent variable models. *Journal of Machine Learning Research*, 11:2001–2049.

Gillenwater, J., Ganchev, K., Graça, Jo a., Pereira, F., and Taskar, B. (2010). Sparsity in dependency grammar induction. In *ACL '10: Proceedings of the ACL 2010 Conference Short Papers*, pages 194–199, Morristown, NJ, USA. Association for Computational Linguistics.

Gold, E. M. (1967). Language identification in the limit. *Information and Control*, 10(5):447–474.

Headden, III, W. P., Johnson, M., and McClosky, D. (2009). Improving unsupervised dependency parsing with richer contexts and smoothing. In *HLT-NAACL*, pages 101–109.

Hsu, D., Kakade, S. M., and Zhang, T. (2009). A spectral algorithm for learning hidden markov models. In *COLT*.

Johnson, M. (1998). Pcfg models of linguistic tree representations. *Comput. Linguist.*, 24(4):613–632.

Johnson, M., Griffiths, T. L., and Goldwater, S. (2006). Adaptor grammars: A framework for specifying compositional nonparametric bayesian models. In *NIPS*, pages 641–648.

Johnson, M., Griffiths, T. L., and Goldwater, S. (2007). Bayesian inference for pcfgs via markov chain monte carlo. In *HLT-NAACL*, pages 139–146.

Klein, D. and Manning, C. D. (2003). Accurate unlexicalized parsing. In *ACL '03: Proceedings of the 41st Annual Meeting on Association for Computational Linguistics*, pages 423–430, Morristown, NJ, USA. Association for Computational Linguistics.

Klein, D. and Manning, C. D. (2004). Corpus-based induction of syntactic structure: Models of dependency and constituency. In *Proceedings of ACL*.

Kumar, M. P., Packer, B., and Koller, D. (2010). Self-paced learning for latent variable models. In Lafferty, J., Williams, C. K. I., Shawe-Taylor, J., Zemel, R., and Culotta, A., editors, *Advances in Neural Information Processing Systems 23*, pages 1189–1197.

Kurihara, K. and Sato, T. (2004). An application of the variational Bayesian approach to probabilistic contextfree grammars. In *IJCNLP-04 Workshop beyond shallow analyses*.

Kurihara, K. and Sato, T. (2006). Variational Bayesian grammar induction for natural language. In *ICGI 2006*, volume 4201 of *LNAI*, pages 84–96.

Langley, P. and Stromsten, S. (2000). Learning context-free grammars with a simplicity bias. In *ECML*, pages 220–228.

Lari, K. and Young, S. (1990). The estimation of stochastic context-free grammars using the inside-outside algorithm. *Computer Speech and Language*, 4:35–36.

Liang, P., Petrov, S., Jordan, M. I., and Klein, D. (2007). The infinite pcfg using hierarchical Dirichlet processes. In *Proceedings of EMNLP-CoNLL*, pages 688–697.

Madeira, S. C. and Oliveira, A. L. (2004). Biclustering algorithms for biological data analysis: A survey. *IEEE/ACM Trans. on Comp. Biol. and Bioinformatics*, 1(1):24–45.

Manning, C. D. and Schütze, H. (1999). *Foundations of statistical natural language processing*. MIT Press, Cambridge, MA, USA.

Marcus, M. P., Santorini, B., and Marcinkiewicz, M. A. (1993). Building a large annotated corpus of english: The penn treebank. *COMPUTATIONAL LINGUISTICS*, 19(2):313–330.

Petrov, S., Barrett, L., Thibaux, R., and Klein, D. (2006). Learning accurate, compact, and interpretable tree annotation. In *ACL-44: Proceedings of the 21st International Conference on Computational Linguistics and the 44th annual meeting of the Association for Computational Linguistics*, pages 433–440, Morristown, NJ, USA. Association for Computational Linguistics.

Poon, H. and Domingos, P. (2009). Unsupervised semantic parsing. In *EMNLP*, pages 1–10.

Poon, H. and Domingos, P. (2011). Sum-product networks : A new deep architecture. In *Proceedings of the Twenty-Seventh Conference on Uncertainty in Artificial Intelligence (UAI)*.

Rohde, D. and Plaut, D. (1999). Language acquisition in the absence of explicit negative evidence: How important is starting small? *Cognition*, 72:67–109.

Smith, N. A. and Eisner, J. (2006). Annealing structural bias in multilingual weighted grammar induction. In *ACL*.

Solan, Z., Horn, D., Ruppin, E., and Edelman, S. (2005). Unsupervised learning of natural languages. *Proc. Natl. Acad. Sci.*, 102(33):11629–11634.

Spitkovsky, V. I., Alshawi, H., and Jurafsky, D. (2010a). From baby steps to leapfrog: How "less is more" in unsupervised dependency parsing. In *NAACL*.

Spitkovsky, V. I., Alshawi, H., Jurafsky, D., and Manning, C. D. (2010b). Viterbi training improves unsupervised dependency parsing. In *Proceedings of the Fourteenth Conference on Computational Natural Language Learning*, CoNLL '10, pages 9–17, Stroudsburg, PA, USA. Association for Computational Linguistics.

Stolcke, A. (1993). Boogie. ftp://ftp.icsi.berkeley.edu/pub/ai/stolcke/software/boogie.shar.Z.

Stolcke, A. and Omohundro, S. M. (1994). Inducing probabilistic grammars by Bayesian model merging. In *ICGI*, pages 106–118.

Tax, D. M. J. (2001). *One-class classification: Concept-learning in the absence of counter-examples*. PhD thesis, Delft University of Technology.

Teh, Y. W., Jordan, M. I., Beal, M. J., and Blei, D. M. (2006). Hierarchical Dirichlet processes. *Journal of the American Statistical Association*, 101(476):1566–1581.

Tu, K. and Honavar, V. (2008). Unsupervised learning of probabilistic context-free grammar using iterative biclustering. In *Proceedings of 9th International Colloquium on Grammatical Inference (ICGI 2008)*, LNCS 5278.

Tu, K. and Honavar, V. (2011). On the utility of curricula in unsupervised learning of probabilistic grammars. In *Proceedings of the Twenty-second International Joint Conference on Artificial Intelligence (IJCAI 2011)*.

Tu, K. and Honavar, V. (2012). Unambiguity regularization for unsupervised learning of probabilistic grammars (under review).

van Zaanen, M. (2000). ABL: Alignment-based learning. In *COLING*.

Zhu, S.-C. and Mumford, D. (2006). A stochastic grammar of images. *Found. Trends. Comput. Graph. Vis.*, 2(4):259–362.